

Math 1303 - Math in the Liberal Arts

Final Exam - 2005.12.05

Due Date - 2005.12.12 8:00 AM

Please answer each question as fully as possible, showing ALL your work and explaining ALL your steps. NO credit will be given for just an answer. Also be sure to write neatly, cleanly and in an orderly fashion.

1. If A and B be are sets such that $n(A) = 250$ and $n(B) = 200$ with $n(A \cap B) = 50$. What is $n(A \cup B)$.

2. If the cardinality of the universal set U is 12 (i.e. $n(U) = 12$), with $n(A \cap B) = n(A \cap C) = n(B \cap C) = 6$, $n(A \cap B \cap C) = 4$ and $n(A \cup B \cup C) = 10$, find $n(A \cup B)$, $n(A' \cup C)$ and $n(A \cap B')$.

3. Construct the truth table for the following symbolic statement:

$$(r \rightarrow \sim q) \rightarrow (\sim p \vee q \leftrightarrow \sim r)$$

4. Determine if the following statements are equivalent.

$$[\sim (p \rightarrow q)] \wedge [\sim (q \rightarrow p)], \quad \sim (p \leftrightarrow q)$$

5. Show that the sum of seven consecutive integers is 7 times the middle number.

6. If y varies inversely as the square of x with the constant of proportionality defined to be k , how does x vary in terms of y , and what is the constant of proportionality?

7. Convert the base 10 number 23789783 to base 4.

8. Perform the following base 7 multiplication:

$$2465_7 \times 143_7$$

9. Show that the following set is infinite by placing it in a 1-to-1 correspondence with \mathbb{N} .

$$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots\right\}$$

10. Determine if the following syllogism is valid or invalid.

The author took out a student loan.

Some people who took out student loans didn't repay them.

\therefore The author didn't repay her student loan.

11. Determine if the following syllogism is valid or invalid.

The author took out a student loan.

All people who took out student loans didn't repay then.

\therefore The author didn't repay her student loan.

12. Factor $210x^2 - 1093x - 770$.

13. Shade the region in the cartesian coordinate plane which satisfies **ALL** of the following inequalities. What shape does the region form?

$$\begin{cases} y + x < 1 \\ y - x < 1 \\ y + x > -1 \\ y - x > -1 \end{cases}$$

14. Shade the region in the cartesian coordinate plane which satisfies **ALL** of the following inequalities. What shape does the region form?

$$\begin{cases} y + x < 1 \\ y - x < 1 \\ y + x > -1 \\ y - x > -1 \\ x > -\frac{3}{4} \\ x < \frac{3}{4} \\ y > -\frac{3}{4} \\ y < \frac{3}{4} \end{cases}$$

15. Determine if the following logical statements are equivalent.

- 1) If $a \cdot b = 0$, then $a = 0$ or $b = 0$.
- 2) If $a \cdot b = 0$ and $a \neq 0$, then $b = 0$.
- 3) If $a \cdot b = 0$ and $b \neq 0$, then $a = 0$.