

Math 1303 - Math in the Liberal Arts

Homework #1 - 2005.08.26

Due Date - 2005.09.02

Solutions

1. Show that the following equivalencies are true:

a) $p \wedge q \Leftrightarrow \sim (\sim p \vee \sim q)$.

We already know the truth table for $p \wedge q$, so we will focus on the truth table for the right hand side, $\sim (\sim p \vee \sim q)$.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim (\sim p \vee \sim q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

Notice that this is indeed the truth table for $p \wedge q$.

b) $p \rightarrow q \Leftrightarrow \sim p \vee q$.

Similarly here, we will work out the truth table for $\sim p \vee q$.

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Once again, this proves the equivalency.

c) $p \leftrightarrow q \Leftrightarrow (\sim p \vee q) \wedge (\sim q \vee p)$

We can use the above result to simplify the computations of the following truth table slightly...Notice once again that the two statements are equivalent.

p	q	$\sim q$	$\sim q \vee p$	$\sim p \vee q$	$(\sim p \vee q) \wedge (\sim q \vee p)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	T	T	T

2. Using the information in problem 1, write the biconditional (\leftrightarrow) in terms of disjunction (\vee) and negation (\sim) only. Notice that this implies that the connectives \wedge , \rightarrow , and \leftrightarrow can be expressed in terms of \vee and \sim only!

Using 1a) and 1c), we see that

$$p \leftrightarrow q \Leftrightarrow \sim (\sim (\sim p \vee q) \vee \sim (\sim q \vee p))$$

3. Let p and q be arbitrary statements, t be a tautology (a statement which is always true), and f be a self-contradiction (a statement which is always false). Determine if the following equivalencies are correct.

a) $p \rightarrow t \Leftrightarrow t$

Correct

b) $f \rightarrow q \Leftrightarrow f$

Incorrect, in fact, one has that $f \rightarrow q \Leftrightarrow t$.

c) $p \vee t \Leftrightarrow t$

Correct

d) $p \wedge f \Leftrightarrow f$

Correct

4. Negate the following statements:

a) There are no extra credit questions.

The quantifier *none are*, whose negation is *some are*, is used in the above statement. Therefore, the negated sentence should read:

Some questions are extra credit questions.

b) Every student will do well in this class.

The quantifier here is *every*, which is the same as *all*. The negation of *all* is *some are not*. The negated sentence is:

Some students will not do well in this class.

c) Some students will get a perfect score on this homework assignment.

The quantifier here is *some*. The negation of *some* is *none*. The negated sentence is:

No student will get a perfect score on this homework assignment.