

Math 1303 - Math in the Liberal Arts

Homework #2 - 2005.08.31

Due Date - 2005.09.09

Solutions

1. Show that $p \rightarrow (q \vee r) \Leftrightarrow (p \wedge \sim q) \rightarrow r$.

| p | q | r | $q \vee r$ | $p \rightarrow (q \vee r)$ | p | q | r | $\sim q$ | $p \wedge \sim q$ | $(p \wedge \sim q) \rightarrow r$ |
|-----|-----|-----|------------|----------------------------|-----|-----|-----|----------|-------------------|-----------------------------------|
| T | T | T | T | T | T | T | T | F | F | T |
| T | T | F | T | T | T | T | F | F | F | T |
| T | F | T | T | T | T | F | T | T | T | T |
| T | F | F | F | F | T | F | F | T | T | F |
| F | T | T | T | T | F | T | T | F | F | T |
| F | T | F | F | T | F | T | F | F | F | T |
| F | F | T | T | T | F | F | T | T | F | T |
| F | F | F | F | T | F | F | F | T | F | T |

Notice from the above truth tables that the two statements are indeed equivalent.

2. Rewrite the following two compound statements in an equivalent fashion using the above identity.

a) If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

If $a \cdot b = 0$ and $a \neq 0$, then $b = 0$.

b) If p is a factor of the product $a \cdot b$ and p is not a factor of a , then p is a factor of b .

If p is a factor of the product $a \cdot b$, then either p is a factor of a or p is a factor of b .

3. Determine if the following arguments are valid or invalid.

First we will define h : *apersonlivesinHonolulu* and o : *apersonlivesonOahu*.

a) Anyone who lives in the city Honolulu, HI also lives on the island of Oahu.

Kanoe lives on the island of Oahu.

Therefore, Kanoe lives in the city Honolulu, HI.

This is in the form:

$$\begin{array}{l}
 h \rightarrow o \\
 o \\
 \text{---} \\
 \therefore h
 \end{array}$$

which is the Fallacy of the Converse. Thus the argument is invalid.

b) Anyone who lives in the city Honolulu, HI also lives on the island of Oahu.

Kanoe does not live on the island of Oahu.

Therefore, Kanoe does not live in the city Honolulu, HI.

This is in the form:

$$\begin{array}{l} h \rightarrow o \\ \sim o \\ \hline \therefore \sim h \end{array}$$

which is the Law of Contraposition. Thus the argument is valid.

c) Anyone who lives in the city Honolulu, HI also lives on the island of Oahu.
Kanoë does not live in the city Honolulu, HI.
Therefore, Kanoë does not live on the island of Oahu.

This is in the form:

$$\begin{array}{l} h \rightarrow o \\ \sim h \\ \hline \therefore \sim o \end{array}$$

which is the Fallacy of the Inverse. Thus the argument is invalid.

d) Anyone who lives in the city Honolulu, HI also lives on the island of Oahu.
Kanoë lives in the city of Honolulu, HI.
Therefore, Kanoë lives on the island of Oahu.

This is in the form:

$$\begin{array}{l} h \rightarrow o \\ h \\ \hline \therefore o \end{array}$$

which is the Law of Detachment. Thus the argument is valid.

4. Determine which of the following statements are equivalent:

- a*: If a number ends in zero, then it is divisible by five.
- b*: If a number is divisible by five, then it ends in zero.
- c*: If a number does not end in zero, then it is not divisible by five.
- d*: If a number is not divisible by five then it does not end in zero.

Notice that if *a* is my original statement, then *b* is the converse, *c* the inverse and *d* the contrapositive. Thus $a \equiv d$ and $b \equiv c$.