

Math 1303 - Math in the Liberal Arts

Homework #3 - 2005.09.02

Due Date - 2005.09.12

Solutions

Consider the following situation:

My stereo system is faulty: there is no sound coming out of the left speaker. Switching the speaker leads will not bring sound to the left speaker if and only if the left speaker is faulty. If switching the speaker leads causes the right speaker to fail, then there is a fault with either the amplifier or the CD player. Switching the leads from the CD player has no effect if and only if there is no problem with the CD player. I discovered the following: switching the leads to the speakers resulted in both channels failing, and switching the leads from the CD player reversed the problem from the left to the right speaker. Therefore replacing the CD player and the left speaker will solve the problem.

Define the following statements:

sl : switching speaker leads results in left channel failure

sr : switching speaker leads results in right channel failure

cl : switching CD leads results in left channel failure

cr : switching CD leads results in right channel failure

fl : fault with left speaker

fa : fault with amplifier

fc : fault with CD player

1. Rewrite the situation above in symbolic form using the statements previously defined.

The following gives the standard form of the above argument:

$$\begin{array}{l} sl \leftrightarrow fl \\ sr \rightarrow (fa \vee fc) \\ cl \leftrightarrow \sim fc \\ sl \wedge sr \wedge cr \\ \hline \therefore (fl \vee fc) \wedge \sim fa \end{array}$$

Notice that the conclusion is given by $(fl \vee fc) \wedge \sim fa$, as opposed to $fc \wedge fl$, as replacing the CD player and the left speaker means the fault was NOT with the amplifier, but IS with either the CD player, the left speaker, or with both the CD player AND the left speaker.

2. Show that if fl , fa , fc , sl , sr , and cr are all true, and cl false, then the statement found in problem 1 is false, making the argument invalid.

The above argument can be expressed as

$$[(sl \leftrightarrow fl) \wedge (sr \rightarrow (fa \vee fc)) \wedge (cl \leftrightarrow \sim fc) \wedge (sl \wedge sr \wedge cr)] \rightarrow ((fl \vee fc) \wedge \sim fa)$$

Substituting in the correct values given in the problem yields the condition $t \rightarrow f$ which is false. Thus the argument is invalid.