Math 2013 - Introduction to Discrete Mathematics

Final Exam - 2005.12.07

Due Date - 2005.12.14 02:00 PM

Please answer each question as fully as possible, showing ALL your work and explaining ALL your steps. NO credit will be given for just an answer. Also be sure to write neatly, cleanly and in an orderly fashion.

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1. Construct a truth table for the following proposition.

$$(p \vee q) \wedge (r \leftrightarrow s) \rightarrow q \wedge \sim p \vee r$$

2. Write the converse, inverse and contrapositive of the following statement.

   If $|x| > a$, then $x < -a$ or $x > a$.

3. Prove or disprove the following equality between sets A and B.

$$[A \cap (A - B)] \cup (A' \cup B')' = A - B$$

4. Write the following defined language L over the alphabet $\Sigma = \{a, b\}$ recursively.

$$L = \{x \in \Sigma^* \mid x \text{ contains only one } a\}$$

5. Let $f : X \rightarrow Y$ and $G : Y \rightarrow Z$. Prove that if f and g are bijective, then $g \circ f$ is bijective.

6. Let A, B and C be square matrices of order 2. Prove that $A(B + C) = AB + AC$.

7. Evaluate the following sum:

$$\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} i \cdot j \cdot k$$

8. Prove that any two consecutive integers are relatively prime.

9. Perform the following base 5 multiplication:

$$43244_5 \times 123232_5$$

10. Let t_n denote the n^{th} triangular number for $n \geq 2$. Prove the following:

$$\sum_{i=1}^{n} t_i = \frac{n(n+1)(n+2)}{6}$$

Hint: There is a formula for the n^{th} triangular number, use it!

11. Define $f(n) = \sum_{i=1}^{n} (2i-1)^2$. Find values of $C, D > 0$ and $n_0 \in \mathbb{N}$ such that the following inequality is satisfied.

$$Cn^3 \leq |f(n)| \leq Dn^3$$

12. Solve the following LNHRRWCC:

$$a_n = 8a_{n-1} - 24a_{n-2} + 32a_{n-3} - 16a_{n-4}, \quad a_0 = 1, a_1 = 4, a_2 = 44, a_3 = 272.$$

13. Let F_n be the n^{th} Fibonacci number. Remember that the recursive definition of the Fibonacci sequence was given by

$$F_n = F_{n-1} + F_{n-2}$$

with $F_1 = F_2 = 1$. In class we showed that the general form of the solution to the above relation was given by

$$f_n = C\left(\frac{1+\sqrt{5}}{2}\right)^n + D\left(\frac{1-\sqrt{5}}{2}\right)^n,$$

where the values for C and D were $\pm\frac{1}{\sqrt{5}}$. Find the constants C and D for the Lucas sequence. Where the recursive relation L_n is the same as that of the Fibonacci sequence F_n, but with initial conditions $L_1 = 1$ and $L_2 = 3$.

14. Let $A = \{0, 1, 2, 3, 4\}$. Compute the number of unique partitions of A.

15. Consider the following sequence: $\{1, 4, 8, 13, 19, 26\}$.
   a) What is the next number in the sequence?
   b) Write the sequence in recursive relation form.
   c) Solve the recurrence relation, and prove that your explicit formula for n^{th} term in the sequence is correct by induction.