Math 2013 - Introduction to Discrete Mathematics

Homework #1 - 2005.08.24

Due Date - 2005.08.31

Solutions

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1. Show that the connectives $\wedge$, $\rightarrow$, and $\leftrightarrow$ can be expressed in terms of $\vee$ and $\sim$ only.

Problem 44 in section 1.2 states that $p \wedge q \equiv\sim (\sim p \vee \sim q)$. Thus conjunction has been written in terms of disjunction and negation. Law 18 on table 1.13 states that $p \rightarrow q \equiv\sim p \vee q$. So conditional is expressed in terms of disjunction and negation. Finally, notice that for the biconditional, one has the following relation which will be proven in problem 2:

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)\,.$$

Using the new formulations of the conditional, we can rewrite the biconditional as:

$$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)\,.$$

Finally, substituting in the new formulation of the conjunction, we arrive at the final form:

$$p \leftrightarrow q \equiv=\sim (\sim (\sim p \vee q) \vee \sim (\sim q \vee p))$$

2. Prove or disprove the following:

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

We will write down a truth table for the right hand side of the equivalency.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Notice that this is indeed the truth table for the biconditional.

3. Prove by contradiction that $\sqrt{2}$ is an irrational number.

We will assume that $\sqrt{2}$ is rational. Thus one can set $\sqrt{2} = \frac{p}{q}$, where m and n are relatively prime integers. Squaring both sides yields the equation $m^2 = 2n^2$. Thus m^2 is even, which implies that m itself must be even (why?). Therefore m^2 is divisible by 4. Thus $2n^2$ must also be divisible by 4, which implies that n must also be even. Hence m and n are not relatively prime. This is a contradiction.

4. Let, a, b and c be arbitrary real numbers. Prove that if $ab = ac$, then either $a = 0$ or $b = c$.

Notice that the form of this proof is $p \rightarrow (q \vee r)$, which is equivalent to $(p \wedge \sim q) \rightarrow r$. It is enough to prove the following statement:

If $ab = ac$ and $a \neq 0$, then $b = c$.

As a result, given the equation $ab = ac$ with $a \neq 0$, one can divide both sides by a, since $a \neq 0$ to prove the statement.

5. Prove or disprove that the function $f(x) = 9x^2 - 471x + 6203$ yields a prime for any nonnegative integer.

It would be hard to prove that the above statement, so one should automatically try to disprove it. If you work hard enough, you can see that the first 40 integers ($x = 0, 1, 2, \ldots 39$) do yield prime numbers. However at $x = 40$, $f(x) = 1763 = 41 \cdot 43$.

6. What can be said about the statement $p \rightarrow q$ if it is known that $\sim p \vee q$ is true?

Using the result of problem 1, it is clear that the two statements in the problem are equivalent. Thus if $\sim p \vee q$ is true, then $p \rightarrow q$ must be true as well.

7. Write the truth table for the following statement:

$$(p \vee q) \wedge (r \rightarrow \sim p) \leftrightarrow \sim q \vee r$$

p	q	r	$p \vee q$	$\sim p$	$r \rightarrow \sim p$	$(p \vee q) \wedge (r \rightarrow \sim p)$	$\sim q$	$\sim q \vee r$	$FINAL$
T	T	T	T	F	F	F	F	T	F
T	T	F	T	F	T	T	F	F	F
T	F	T	T	F	F	F	T	T	F
T	F	F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	F	T	T
F	T	F	T	T	T	T	F	F	F
F	F	T	F	T	T	F	T	T	F
F	F	F	F	T	T	F	T	T	F