Math 2013 - Introduction to Discrete Mathematics

Homework #2 - 2005.08.31

Due Date - 2005.09.09

Solutions

---

1. Let x and y be real numbers, and $P(x, y) : x^2 + y^2 = 1$. Determine if the following statements are true or false. Note: $\exists!$ means '*there exists a unique*'.

a) $(\forall y)\,(\exists x)\,P(x, y)$

This statement says that given a y, one can find an x such that $x^2 + y^2 = 1$. This is false, since if $|y| > 1$, no such x exists.

b) $(\exists y)\,(\forall x)\,P(x, y)$

This statement says that there is a y, such that for any x one has $x^2 + y^2 = 1$. This is false, since no one value of y will work for all x.

c) $(\exists y)\,(\exists! x)\,P(x, y)$

This says there is a y such that there is a unique x so that $x^2 + y^2 = 1$. Notice that if $y = \pm 1$, then there is only one value of x which will satisfy $x^2 + y^2 = 1$ (here $x = 0$).

2. Let $\Sigma_1 = \{0, 10\}$ and $\Sigma_2 = \{0, 010\}$ be alphabets and define Σ_1^* and Σ_2^* to be the set of words over Σ_1 and Σ_2 respectively. Prove the following:

a) $\Sigma_2^* \subseteq \Sigma_1^*$

All that is required is for one to be able to write any word in Σ_2^* in terms of the alphabet of Σ_1. This is trivial since $010_2 = 0_1 10_1$. Here the subscripts denote which alphabet the symbols came from.

b) $\Sigma_1^* \not\subseteq \Sigma_2^*$

This is true since $10 \in \Sigma_1^*$ and $10 \notin \Sigma_2^*$.

3. Define $U = \{0, 1, 2, 3, a, b, c, d\}$, $A = \{0, 1, 2, 3\}$, $B = \{a, b, c, d\}$ and $C = \{2, 3, a\}$. Find the binary representations of the following sets.

First we will define U as follows:

	d	c	b	a	3	2	1	0
$U =$	1	1	1	1	1	1	1	1

Then

$A =$	0	0	0	0	1	1	1	1
$B =$	1	1	1	1	0	0	0	0
$C =$	0	0	0	1	1	1	0	0

a) $B \cup C =$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

b) $A \cap C =$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |

c) $A \oplus C =$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |

d) $C - A =$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

e) $U - C =$ | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |

4. Create a finite partition of $\mathbb{R}$.

As a simple example, consider $S_1 = (-\infty, 0]$ and $S_2 = (0, \infty)$. Clearly $S_1 \cup S_2 = \mathbb{R}$, $S_1 \cap S_2 = \emptyset$, and $S_i \neq \emptyset$ for $j = 1, 2$.

5. Create an infinite partition of $\mathbb{R}$.

Here, define $Sj = (j, j+1]$ for $j \in I = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$. Notice that

$$\bigcup_{j \in I} S_j = \mathbb{R},$$

and $S_j \cap S_i = \emptyset \quad \forall i, j \in I \text{s.t.} j \neq i$ and also $S_j \neq \emptyset \quad \forall j \in I$. It should be noted that this is only one partition out of an infinite number of possible partitions.

6. Compute the power set of the set $S = \{0, 1, 101\}$

$$\mathcal{P}(S) = \{\emptyset, \{0\}, \{1\}, \{101\}, \{0,1\}, \{0,101\}, \{1,101\}, \{0,1,101\}\}$$

7. Let $U = a, b, c, d, e, f, g$. Using the binary representation of sets, answer the following questions. Do not write your answers in binary!

a) Find the subset of the set U which follows the set represented by: | 1 | 0 | 0 | 1 | 1 | 1 | 1 |

The set which follows the one above can be written in binary form as:

1	0	1	0	0	0	0

which corresponds to the set $Y = \{e, f\}$.

b) Find the subset of the set U which is followed by the set represented by: | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
The set which precedes the above set would be written as:

1	0	1	1	0	1	1

which corresponds to the set $M = \{a, b, d, e, g\}$