

Math 2013 - Introduction to Discrete Mathematics

Homework #3 - 2005.09.14

Due Date - 2005.09.19

1. Show that if n is an odd integer, then

$$\lceil \frac{n^2}{4} \rceil = \frac{n^2 + 3}{4}$$

2. Let $\max\{x, y\}$ denote the maximum of x and y and $\min\{x, y\}$ denote the minimum of x and y . Prove the following two statements:

a) $\max\{x, y\} + \min\{x, y\} = x + y$

b) $\max\{x, y\} - \min\{x, y\} = |x - y|$

3. Prove that a bijection exists between any two closed intervals $[a, b]$ and $[c, d]$ where $a < b$ and $c < d$. *Hint: find a formula that works!*

4. Let $\Sigma = \{a, ba, abb, bbb\}$ be an alphabet, and define Σ^* as usual. Is $f(x) = \|x\|$, where $x \in \Sigma^*$ a function? Why or why not?

5. Compute the domain and range of the following functions.

a) $f(x) = x^2 - 1$

b) $g(x) = \frac{1}{x}$

c) $h(x) = (f + g)(x)$

d) $h(x, y) = f(x) + g(y)$

e) $h(x, y) = (f(x), g(y))$