

# Math 2013 - Introduction to Discrete Mathematics

Homework #4 - 2005.10.19

Due Date - 2005.10.28

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1. Prove the following by induction:

$$\sum_{k=1}^n k^2 = \frac{1}{6}n(n+1)(2n+1)$$

2. Prove the following by induction:

$$\sum_{k=0}^n 2^k = 2^{n+1} - 1$$

3. Show the following:

$$\sum_{k=1}^{2n} (-1)^k \cdot k = n$$

*Hint:* If you use the fact that

$$\sum_{k=1}^n k = \frac{n(n+1)}{2},$$

you will not have to use induction!

4. Given that  $x_k \in \mathbb{R}$  for  $k = 1, \dots, n \geq 2$ , prove the following by induction:

$$\left| \sum_{k=1}^n x_k \right| \leq \sum_{k=1}^n |x_k|$$

*Hint:* You might need the following facts:  $-|a| \leq a \leq |a|$  and  $|a| \leq b \Leftrightarrow -b \leq a \leq b$  for all  $a, b \in \mathbb{R}$ .