Math 2013 - Introduction to Discrete Mathematics

Quiz #11 - 2005.10.26

Solutions

Prove the following by the Principle of Mathematical Induction:

$$\sum_{k=1}^{n} 3k^2 - k - 2 = n(n+2)(n-1), \quad n \geq 1$$

Notice that when $n = 1$, the above holds true (i.e. $0 = 0$).
Next, we will assume that it hold true for n. We need to show then that it will hold true for $n+1$. I.e. we must show:

$$\sum_{k=1}^{n+1} 3k^2 - k - 2 = n(n+3)(n+1)$$

To do this, we first write

$$\begin{aligned}\sum_{k=1}^{n+1} 3k^2 - k - 2 =& \sum_{k=1}^{n+1} 3k^2 - k - 2 + 3\left(n+1\right)^2 - (n+1) - 2 \\ =& n(n+2)(n-1) + 3\left(n+1\right)^2 - (n+1) - 2 \\ =& n(n+2)(n-1) + 3n^2 + 5n \\ =& n(n+2)(n-1)n(3n+5) \\ =& n\left(n^2 + 4n + 3\right) \\ =& n(n+3)(n+1)\end{aligned}$$

This proves that

$$\sum_{k=1}^{n} 3k^2 - k - 2 = n(n+2)(n-1)$$