

Math 2013 - Introduction to Discrete Mathematics

Quiz #13 - 2005.11.11

Solutions

Consider the following ordered array which contains 22 elements:

1	2	4	4	4	5	6	7	9	22	24	34	46	47	58	58	58	59	60	61	62	63
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1. How many steps will it take to find $key = 60$ if one uses the binary search algorithm proposed in the book? Be sure to justify your answer.

First time, $low = 1$, $high = 22$, and thus $mid = \lfloor \frac{22+1}{2} \rfloor = 11$ with $x_{11} = 24 < 60$.

So now $low = mid + 1 = 12$, $high = 22$ and $mid = 17$ with $x_{17} = 58 < 60$.

So now $low = mid + 1 = 18$, $high = 22$ and $mid = 20$ with $x_{20} = 61 > 60$.

So now $low = 18$, $high = mid - 1 = 19$ and $mid = 18$ and $x_{18} = 59 < 60$.

Finally, $low = mid = high = 19$ and $x_{19} = 60$.

Notice that it took 5 steps to find key .

2. Consider the algorithm to compute the GCD of two positive integers x and y given below (very similar to the one discussed in class).

```
x = 348,   y = 168
counter = 0
while x ≠ y do
    counter = counter + 1
    if x > y then
        x = x - y
    else
        y = y - x
end while
gcd = x
```

What is the final value of $counter$ and what is the value of gcd ?

The following is a table of the values of *counter*, *x* and *y*.

counter	x	y
0	348	168
1	180	168
2	12	168
3	12	156
4	12	144
5	12	132
6	12	120
7	12	108
8	12	96
9	12	84
10	12	72
11	12	60
12	12	48
13	12	36
14	12	24
15	12	12