

# Math 2013 - Introduction to Discrete Mathematics

Quiz #15 - 2005.12.05

Solutions

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Consider the recursive relation  $a_n = -4a_{n-1} + 21a_{n-2}$ ,  $n \geq 0$ .

1. Find a formula for the general solution.

Setting  $a_n = \alpha^n$  gives the quadratic equation  $\alpha^2 + 4\alpha - 21 = 0$ . This can be factored as  $(\alpha + 7)(\alpha - 3)$ . This the solutions are  $\alpha = -7$  and  $\alpha = 3$ . The general form of the solution is:

$$a_n = C \cdot 3^n + D \cdot (-7)^n$$

2. Find the solution if  $a_0 = 5$  and  $a_1 = -15$ .

Setting  $a_0 = 5$  and  $a_1 = -15$  gives the system

$$\begin{cases} C + D = 5 \\ 3C - 7D = -15 \end{cases}$$

Solving this system gives  $C = 2$  and  $D = 3$ . Thus the solution is

$$a_n = 2 \cdot 3^n + 3 \cdot (-7)^n$$

3. If someone asks you ‘What floats on water?’, how might you respond?

Apples  
Very small rocks  
Cider  
Gravy  
Cherries  
Mud  
Churches  
Lead  
A duck