Math 2013 - Introduction to Discrete Mathematics

Quiz #5 - 2005.09.21

Solutions

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1. Determine if the following function is injective and/or surjective on the specified domain and codomain.

   $f : \mathbb{R} \to \mathbb{Z}$ where $f(x) = \lceil x \rceil$

   For f to be surjective, one must show that $\forall z \in \mathbb{Z}$, $\exists x \in \mathbb{R}$ such that $f(x) = z$. Clearly this will hold true as given any integer z, there is a real number (in fact many real numbers!) that when rounded up to the nearest integer, will in fact be z.

   Notice that f is not injective. As an example, given any two distinct real numbers $x_1, x_2 \in (0, 1)$, then $f(x_1) = f(x_2) = 1$, with $x_1 \neq x_2$.

2. Determine if the following function is injective and/or surjective on the specified domain and codomain.

   $g : \mathbb{N} \to \mathbb{N}$ where $f(x) = x + 1$

   To see that g is injective, notice that if $g(x_1) = g(x_2)$, then $x_1 + 1 = x_2 + 1$. This means that $x_1 = x_2$, and injectivity is proven.

   There is no $x \in \mathbb{N}$ s.t. $g(x) = 1$. Thus the value of 1, in the codomain of g is not mapped to, making the function not surjective.