Math 2143 - Brief Calculus with Applications

Homework #4 - 2006.02.20

Due Date - 2006.02.27

Solutions

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1. Let $f(x) = \sqrt{1+x}$ and $g(x) = x^2$. Answer the following questions.

a) What is the domain of $f(x)$?

Domain is $[-1, \infty)$

b) What is the range of $f(x)$?

Range is $[0, \infty)$

c) What is the domain of $g(x)$?

Domain is $\mathbb{R}$

d) What is the range of $g(x)$?

Range is $[0, \infty)$

e) What is the domain of $f \circ g(x)$?

The domain of $f \circ g(x)$ is $\mathbb{R}$

f) What is the range of $f \circ g(x)$?

The range of $f \circ g(x)$ is $[1, \infty)$

g) What is the domain of $g \circ f(x)$?

The domain of $g \circ f(x)$ is $[-1, \infty)$

h) What is the range of $g \circ f(x)$?

The range of $g \circ f(x)$ is $[0, \infty)$.

2. Let $f(x) = \frac{1}{\sqrt{3-x}}$ and $g(x) = x^2 - 1$. Answer the following questions.

a) What is the domain of $f(x)$?

Domain is $(-\infty, 3)$

b) What is the range of $f(x)$?

Range is $(0, \infty)$

c) What is the domain of $g(x)$?

Domain is $\mathbb{R}$

d) What is the range of $g(x)$?

Range is $[-1, \infty)$

e) What is the domain of $f \circ g(x)$?

The domain requires more work in this case. We must make sure that no matter what values of x we plug into g, that the result lies in the domain of f (why?). Therefore, we are looking for all values of x such that $g(x)$ is on the interval $(-\infty, 3)$. Thus we require that $x^2 - 1 < 3$ or $-\sqrt{4} < x < \sqrt{4}$, i.e. the domain is $(-2, 2)$.

f) What is the range of $f \circ g(x)$?

Clearly as x gets close to ± 2 the function goes to ∞. The minimum value would be when the denominator is largest. This occurs when $x = 0$, which implies that the range is $\left[\frac{1}{2}, \infty\right)$.

g) What is the domain of $g \circ f(x)$?

The domain here is much simpler, as the domain of g is already $\mathbb{R}$, therefore, plugging any value in the domain of f will give a real number, thus the domain is $(-\infty, 3)$.

h) What is the range of $g \circ f(x)$?

Since the domain of $f(x)$ gives a range of $(0, \infty)$, plugging this range into domain of $g(x)$ gives a range of $(-1, \infty)$.