Math 2143 - Brief Calculus with Applications

Homework #6 - 2006.03.23

Due Date - 2006.03.31

Solutions

Using the following function,

$$f(x) = \sqrt{x^2 + \frac{1}{x^2}},$$

answer the following questions.

1. What is the domain of $f(x)$?

   The domain of $f(x)$ is all real numbers except $x = 0$.

2. Does $f(x)$ have any roots? If so, state them.

   Notice that $f(x)$ is strictly positive, thus it has no roots.

3. Does $f(x)$ have any vertical or horizontal asymptotes?

   There is a vertical asymptote at $x = 0$. There are no horizontal asymptotes.

4. The function has two oblique asymptotes. What are they?

   The oblique asymptotes are $y = \pm x$.

5. Compute $f'(x)$. (*Hint: simplify your answer before going on.*)

$$f'(x) = \frac{1}{2\sqrt{x^2 + \frac{1}{x^2}}}\left(2x - 2\frac{1}{x^3}\right) = \frac{1}{\sqrt{x^2 + \frac{1}{x^2}}}\left(\frac{x^4 - 1}{x^3}\right).$$

6. Find all critical points for $f(x)$.

   Critical points are at $x = \pm 1$.

7. Find the intervals of increase and decrease for $f(x)$.

   Notice that the term

$$\frac{1}{\sqrt{x^2 + \frac{1}{x^2}}}$$

is always positive. Thus, to determine the sign of $f'(x)$, all we have to do is look at the term

$$\frac{x^4 - 1}{x^3}.$$

This gives that the function is decreasing on $(-\infty, -1)$ and $(0, 1)$ while increasing on $(-1, 0)$ and $(1, \infty)$.

8. Compute $f''(x)$. (*Hint: simplify your answer before going on.*)

$$\begin{aligned} f''(x) &= \left(1+\frac{3}{x^4}\right)\left(x^2+\frac{1}{x^2}\right)^{-\frac{1}{2}} - \frac{1}{2}\left(\frac{x^4-1}{x^3}\right)\left(2x-\frac{2}{x^3}\right)\left(x^2+\frac{1}{x^2}\right)^{-\frac{3}{2}} \\ &= 2\frac{3x^4+1}{x^6}\frac{1}{\left(x^2+\frac{1}{x^2}\right)^{\frac{3}{2}}} \end{aligned}$$

9. Find the intervals of concavity for $f(x)$.

Notice that $f''(x) > 0$ for all x in the domain of $f(x)$. Therefore, the function is concave up everywhere.

10. Graph $f(x)$ using all the information you have gathered from the above 9 problems.