Math 2143 - Brief Calculus with Applications

Quiz #9 - 2006.03.10

Solutions

Consider the function $f(x) = x^3 - 6x^2 - 135x$. Answer the following questions.

1. Find all the roots of $f(x)$.

   Notice that $f(x)$ can be factored as $f(x) = x(x-15)(x+9)$, therefore the roots are $x = 0$, $x = 15$ and $x = -9$.

2. Find $f'(x)$.

$$f'(x) = 3x^2 - 12x - 135 = 3(x-9)(x+5).$$

3. Find all the critical points of $f(x)$.
   Critical points are at $x = 9$ and $x = -5$.

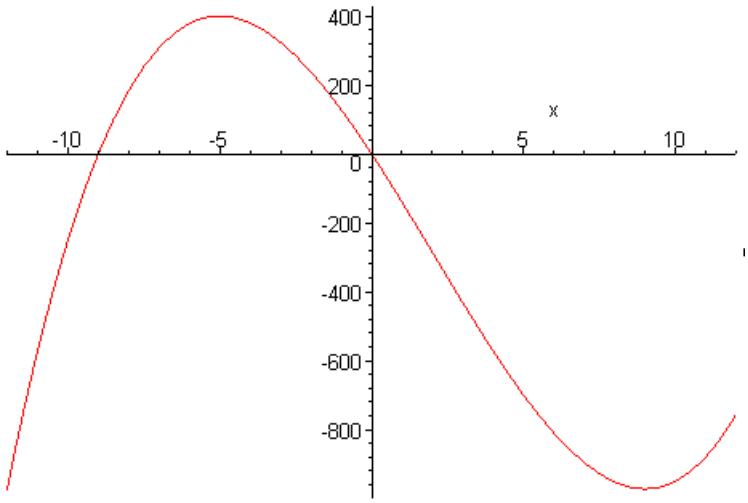
4. State the intervals of increase and decrease of $f(x)$.
   Since $f'(x)$ is a parabola opening upwards with roots given in the previous problem. The intervals of increase are $(-\infty, -5)$ and $(9, \infty)$. The interval of decrease is $(-5, 9)$.

5. Find $f''(x)$.

$$f''(x) = 6x - 12 = 6(x-2).$$

6. State the intervals of concavity of $f(x)$.
   Since $f''(x)$ is just a line, we have that $f(x)$ is concave up on $(2, \infty)$ and concave down on $(-\infty, 2)$.

7. Find all the relative extrema of $f(x)$.
   Since $f'(9) = 0$ and $f''(9) > 0$, we have a relative minimum at $x = 9$. A similar argument for $x = -5$ shows that it is a relative maximum.

8. Find all the inflection points of $f(x)$.
   The only inflection point is at $x = 2$.

9. Sketch a graph of $f(x)$.