

Math 2215 - Calculus 1

Final Exam - 2005.12.

Due Date - 2005.12.12 11:00 AM

Please answer each question as fully as possible, showing ALL your work and explaining ALL your steps. NO credit will be given for just an answer. Also be sure to write neatly, cleanly and in an orderly fashion.

1. Show $|\sin(x) - \cos(x)| \leq \sqrt{2}$.
2. Find the highest and lowest points on $x^2 + xy + y^2 = 12$.
3. Determine all values of a such that $f(x)$ given below has no critical numbers.

$$f(x) = (a^2 + a - 6) \cos(2x) + (a - 2)x + \cos(1)$$

4. Find the n^{th} derivative of $f(x) = \frac{x^n}{(1-x)}$.
5. Compute the following limit:

$$\lim_{x \rightarrow \pi} \frac{e^{\sin(x)} - 1}{x - \pi}$$

6. For what values of a is it true that $a^x \geq 1 + x$ for all x .
7. Consider

$$y = \frac{x}{\sqrt{a^2 - 1}} - \frac{2}{\sqrt{a^2 - 1}} \tan^{-1} \left(\frac{\sin(x)}{a + \sqrt{a^2 - 1} + \cos(x)} \right).$$

Show that

$$y' = \frac{1}{a + \cos(x)}.$$

8. Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{|2x - 1| - |2x + 1|}{x}$$

9. Prove the following inequality:

$$\frac{1}{17} \leq \int_1^2 \frac{1}{1+x^4} dx \leq \frac{7}{24}$$

10. Given that $f(x)$ satisfies the following equation and is differentiable for all x , find $f(x)$.

$$\int_0^x f(t)dt = [f(x)]^2$$

11. Evaluate the following limit:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} \frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n}} \frac{1}{\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}} \frac{1}{\sqrt{n+n}} \right)$$

Hint: Factor $\frac{1}{n}$ out of each term and consider the function $\frac{1}{\sqrt{1+x}}$ on the interval $[0, 1]$.

12. If $x \sin(\pi x) = \int_0^{x^2} f(t)dt$, find $f(4)$.

13. Find the interval $[a, b]$ for which $\int_a^b 2 + x - x^2 dx$ is a maximum.

14. Determine the value of a such that:

$$\lim_{x \rightarrow \infty} \left(\frac{x+a}{x-a} \right)^x = e.$$

15. Prove the following:

$$\int_a^b f'(g(h(x) + i(x))) \cdot g'(h(x) + i(x)) (h'(x) + i'(x)) dx = \int_{g(h(a)+i(a))}^{g(h(b)+i(b))} f'(u) du$$