Math 2215 - Calculus 1

Quiz #11 - 2005.11.09

Solutions

Consider the function $f(x) = \frac{x}{x^2-1}$. Answer the following questions.

1. State the domain of $f(x)$.

   Domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

2. Find all the roots of $f(x)$.

   The only root is at $x = 0$ and $f(0) = 0$.

3. Compute $f'(x)$.

$$f'(x) = \frac{x^2 - 1 - x \cdot 2x}{(x^2-1)^2} = -\frac{x^2+1}{(x^2-1)^2}$$

4. Find <u>all</u> the critical numbers of $f(x)$.

   Since the numerator of $f'(x)$ is never zero, the only critical numbers are when the derivative does not exist. This occurs when $x = \pm 1$.

5. Determine the intervals of increase and decrease of $f(x)$.

   Notice that the derivative is always negative, thus $f(x)$ is decreasing everywhere on its domain.

6. Compute $f''(x)$.

$$f''(x) = -\frac{2x \cdot (x^2-1)^2 - (x^2+1) \cdot 2(x^2-1) \cdot 2x}{(x^2-1)^4} = -\frac{2x \cdot (x^2+3)}{(x^2-1)^4}$$

7. Determine the intervals of concavity for $f(x)$.

   Upon examining the second derivative, it is clear that the sign of concavity changes at $x = -1$, $x = 0$ and at $x = 1$. Thus, we have broken up the number line into 4 intervals. Notice that for $x < -1$, $f''(x) < 0$, for $-1 < x < 0$, $f''(x) > 0$, for $0 < x < 1$, $f''(x) < 0$ and for $x > 1$, $f''(x) > 0$. So $f(x)$ is concave down on $(-\infty, -1) \cup (0, 1)$ and concave up on $(-1, 0) \cup (1, \infty)$.

8. Find the inflection points of $f(x)$ based upon your answer to problem 7.

   There is an inflection point at $x = 0$. One would like to state that inflection points exist at $x = \pm 1$ as well, however the function does not exist at $x = \pm 1$.

9. Does $f(x)$ have any horizontal asymptotes? If so, where?

   We compute the following limit:

$$\lim_{x \to \pm\infty} f(x) = 0.$$

This implies that $y =$ is a horizontal asymptote (and the only one).

10. Compute the following limits:

$$\lim_{x \to -1^-} f(x) = -\infty$$

$$\lim_{x \to -1^+} f(x) = \infty$$

$$\lim_{x \to 1^-} f(x) = -\infty$$

$$\lim_{x \to 1^+} f(x) = \infty$$

11. Use your answers from problems 1-10 to graph $f(x)$.