

Math 2215 - Calculus 1

Quiz #8 - 2005.09.28

Solutions

Compute the second derivative $\left(\frac{d^2y}{dx^2}\right)$ of the implicitly defined function found below. Your final answer should be a function of x and y only.

$$\cos(xy) = xy^2$$

Computing one derivative, we have:

$$-y \sin(xy) \left(y + x \frac{dy}{dx}\right) = y^2 + 2xy \frac{dy}{dx}$$

Solving for $\frac{dy}{dx}$ gives:

$$\frac{dy}{dx} = -\frac{y \sin(xy) + y^2}{x \sin(xy) + 2xy}$$

Taking one more derivative gives:

$$\begin{aligned} \frac{d^2y}{dx^2} = & - \left[\left(\frac{dy}{dx} \sin(xy) + y \cos(xy) \left(y + x \frac{dy}{dx} \right) + 2y \frac{dy}{dx} \right) (x \sin(xy) + 2xy) \right. \\ & \left. - (y \sin(xy) + y^2) \left(\sin(xy) + \cos(xy) \left(y + x \frac{dy}{dx} \right) + 2y + 2x \frac{dy}{dx} \right) \right] / (x \sin(xy) + 2xy)^2 \end{aligned}$$

Substituting in the form of $\frac{dy}{dx}$ into the above expression gives the desired result.