Math 2215 - Calculus 1

Homework #15 - 2007.04.18

Due Date - 2007.04.25

Solutions

Compute the following integrals:

1.

$$\int \frac{\ln\left(\tan^{-1}(x)\right)}{(1+x^2)\tan^{-1}(x)}dx$$

Setting $u = \tan^{-1}(x)$, we have $du = \frac{dx}{1+x^2}$ so

$$\int \frac{\ln\left(\tan^{-1}(x)\right)}{\tan^{-1}(x)\,(1+x^2)}dx = \int \frac{\ln(u)}{u}du.$$

Setting $v = \ln(u)$, then $dv = \frac{1}{u}du$ so

$$\int \frac{\ln(u)}{u}du = \int v\ dv = \frac{1}{2}v^2 + C.$$

Upon resubstitution, we have

$$\int \frac{\ln\left(\tan^{-1}(x)\right)}{(1+x^2)\tan^{-1}(x)}dx = \frac{1}{2}\left(\ln\left(\tan^{-1}(x)\right)\right)^2 + C.$$

2.

$$\int \frac{1}{\sqrt{x}+\sqrt[4]{x}}dx$$

Let's set $u = \sqrt[4]{x}$, we have $dt = \frac{1}{4}x^{-\frac{3}{4}}dx = \frac{1}{4x^{\frac{3}{4}}}dx$. Solving for x in the $u = \sqrt[4]{x}$ equation $x = u^4$. So this also gives $dx = 4u^3du$ and

$$\int \frac{1}{\sqrt{x}+\sqrt[4]{x}}dx = 4\int \frac{u^3}{u^2+u}du.$$

We now do some simplification, first there is a factor of u that we can divide out of everything:

$$4\int \frac{u^3}{u^2+u}du = 4\int \frac{u^2}{u+1}du.$$

Next we long divide out the new integral to get

$$4\int \frac{u^2}{u+1}du = 4\int u - 1 + \frac{1}{u+1}\ du.$$

This is a very doable integral:

$$4\int u - 1 + \frac{1}{u+1}\, du = 4\left(\frac{1}{2}u^2 - u + \ln(u+1) + C\right).$$

Resubstituting $u = \sqrt[4]{x}$ gives

$$4\left(\frac{1}{2}u^2 - u + \ln(u+1) + C\right) = 4\left(\frac{1}{2}\sqrt{x} - \sqrt[4]{x} + \ln(\sqrt[4]{x}+1) + C\right),$$

so

$$\int \frac{1}{\sqrt{x}+\sqrt[4]{x}}dx = 2\sqrt{x} - 4\sqrt[4]{x} + 4\ln(\sqrt[4]{x}+1) + C.$$