

Math 2283 - Introduction to Logic

Homework #10 - 2006.11.15

Due Date - 2006.11.20

Solutions

Consider the wff $(\exists x\forall yGxy \Rightarrow \forall x\exists yGxy)$. Answer the following questions.

1. Given a domain D consisting of only two objects a and b , define the truth values of the unknown predicate G to be:

$$\begin{cases} Gab = T \\ Gaa = T \\ Gba = F \\ Gbb = F \end{cases}$$

Argue that such a structure would invalidate the wff.

Notice that $\forall yGay$ is true (i.e. $x = a$ works), yet $\exists yGby$ is false since Gba and Gbb are both false (i.e. one cannot find a y which makes Gby true), therefore the wff is now in the form $T \Rightarrow F$ which yields a false result.

2. Using the idea above, come up with a structure with only 2 objects in the domain, and a predicate G which invalidates the wff. (Hint: Remember that although G is single letter which represents a two place predicate which involves two objects x and y , your predicate does not have to be limited to one simple equation or sentence!)

Consider $D = \{1, 2\}$ and Gxy is $((x < y) \vee x = 1)$. We will go through each possibility real quick:

$G11$: $((1 < 1) \vee 1 = 1)$ is true

$G12$: $((1 < 2) \vee 1 = 1)$ is true

$G21$: $((2 < 1) \vee 2 = 1)$ is false

$G22$: $((2 < 2) \vee 2 = 1)$ is false

Notice that $G1y$ is true for all y yet one cannot find a y which makes $G2y$ true.

3. Using problem 1 and your answer to problem 2, come up with a structure which has a domain of more than 2 objects which invalidates the wff.

Consider $D = \{1, 2, 3, \dots, n\}$ and Gxy is $((x < y) \vee x = 1)$. Notice that $G1y$ is $\forall y((1 < y) \vee 1 = 1)$ which is always true, however Gny is always false since $N < y$ is always false for all y in the domain and $n = 1$ is also false since $n \neq 1$. Thus $\exists y((n < y) \vee n = 1)$ is false.