

# Math 2283 - Introduction to Logic

Homework #3 - 2006.09.11

Due Date - 2006.09.18

Name: \_\_\_\_\_

---

We are familiar with functions of one variable. An example is  $f(x) = x^2$ , where for instance  $f(2) = 4$ . Consider a function of two variables,  $f(x, y) = x + y - 1$ . Here, we have to input two values, one for  $x$  and one for  $y$ . As an example,  $f(2, 3) = 2 + 3 - 1 = 4$  and  $f(1, -2) = 1 - 2 - 1 = -2$ . So, consider the following functions:

$$f(x, y) = x + y - 1, \quad g(x, y) = x^2y - x$$

Determine if the following logical statements are true or false given the definitions of  $f(x, y)$  and  $g(x, y)$  above.

1.  $\exists x \forall y f(x, y) = 0$

2.  $\exists x \exists y f(x, y) = 0$

3.  $\forall x \exists y f(x, y) = 0$

4.  $\forall x \forall y f(x, y) = 0$

$$5. \exists x \forall y g(x, y) = 0$$

$$6. \exists x \exists y g(x, y) = 0$$

$$7. \forall x \exists y g(x, y) = 0$$

$$8. \forall x \forall y g(x, y) = 0$$

Determine whether or not the following arguments are valid or invalid.

9.

$$p \vee q \Rightarrow r$$

$$r \vee q$$

$$\therefore q \Rightarrow p$$

10.

$$\begin{aligned} p \vee q &\Rightarrow r \\ r \wedge q & \\ \therefore q &\Rightarrow p \end{aligned}$$

11.

$$\begin{aligned} p \vee q &\Rightarrow r \\ r \wedge q & \\ \therefore \sim r &\Rightarrow p \end{aligned}$$

12.

$$\begin{aligned} p \vee q &\Rightarrow r \\ r \vee q & \\ \therefore r & \end{aligned}$$