Math 2283 - Introduction to Logic

Homework #3 - 2006.09.11

Due Date - 2006.09.18

Solutions

We are familiar with functions of one variable. An example is $f(x) = x^2$, where for instance $f(2) = 4$. Consider a function of two variables, $f(x, y) = x + y - 1$. Here, we have to input two values, one for x and one for y. As an example, $f(2, 3) = 2 + 3 - 1 = 4$ and $f(1, -2) = 1 - 2 - 1 = -2$. So, consider the following functions:

$$f(x, y) = x + y - 1, \quad g(x, y) = x^2 y - x$$

Determine if the following logical statements are true or false given the definitions of $f(x, y)$ and $g(x, y)$ above.

1. $\exists x \ \forall y \ f(x, y) = 0$

This statement is false, one cannot find an x which works for all y.

2. $\exists x \ \exists y \ f(x, y) = 0$

This statement is true. One can find an x and a y such that $f(x, y) = 0$. As an example, $f(0, 1) = 0$.

3. $\forall x \ \exists y \ f(x, y) = 0$

This statement is true. For each x, one can find a y such that $f(x, y) = 0$. In particular, consider that for a fixed x, then $y = 1 - x$ gives $f(x, 1-x) = x+1-x-1 = 0$.

4. $\forall x \ \forall y \ f(x, y) = 0$

This statement is false. For each x and for each y, $f(x, y)$ is not always zero.

5. $\exists x \ \forall y \ g(x, y) = 0$

This statement is true, since $g(0, y) = 0$ works for all y.

6. $\exists x\ \exists y\ g(x,y) = 0$

This is true since problem 5 is true (i.e. y can be anything).

7. $\forall x\ \exists y\ g(x,y) = 0$

For each x, one can find a y such that $g(x,y) = 0$. In particular, if $x = 0$ any y will do. If $x \neq 0$, then let $y = \frac{1}{x}$, then notice that $g\left(x, \frac{1}{x}\right) = x^2\frac{1}{x} - x = x - x = 0$.

8. $\forall x\ \forall y\ g(x,y) = 0$

This is false, since $g(x,y) = 0$ does not hold for all x and all y.

Determine whether or not the following arguments are valid or invalid.
9.

$$p \vee q \Rightarrow r$$
$$r \vee q$$
$$\therefore q \Rightarrow p$$

p	q	r	$p \vee q$	$p \vee q \Rightarrow r$	$r \vee q$	$q \Rightarrow p$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	**T**	**T**	**F**
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	F	T

The argument is invalid due to row 5.

10.

$$p \vee q \Rightarrow r$$
$$r \wedge q$$
$$\therefore q \Rightarrow p$$

p	q	r	$p \vee q$	$p \vee q \Rightarrow r$	$r \wedge q$	$q \Rightarrow p$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	F	T
T	F	F	T	F	F	T
F	T	T	T	**T**	**T**	**F**
F	T	F	T	F	F	F
F	F	T	F	T	F	T
F	F	F	F	T	F	T

The argument is invalid due to row 5.

11.

$$p \vee q \Rightarrow r$$
$$r \wedge q$$
$$\therefore \sim r \Rightarrow p$$

p	q	r	$p \vee q$	$p \vee q \Rightarrow r$	$r \wedge q$	$\sim r \Rightarrow p$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	F	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	F
F	F	T	F	T	F	T
F	F	F	F	T	F	F

The argument is valid.

12.

$$p \vee q \Rightarrow r$$
$$r \vee q$$
$$\therefore r$$

p	q	r	$p \vee q$	$p \vee q \Rightarrow r$	$r \vee q$	r
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	F	T	T	T
F	F	F	F	T	F	F

The argument is valid.