Math 2283 - Introduction to Logic

Quiz #9 - 2006.10.16

Solutions

Determine if the following argument is valid or invalid. Do not use a truth table to determine your answer. If the argument is valid, state why (be sure to cover all possibilities). If the argument is invalid, state why and also state the conditions on the atomic wff's which make it invalid.
$((q \wedge r) \Rightarrow s)$
$((p \wedge r) \Leftrightarrow (p \vee q))$
$(s \Rightarrow (r \vee p))$
$(s \vee p)$
$\therefore (p \vee (q \Leftrightarrow r))$

There are two ways to make the conclusion false. They are:
1. $p = F$, $q = T$ and $r = F$
2. $p = F$, $q = F$ and $r = T$

Consider the first case. We will attempt to make all of the premises true to invalidate the argument. To make the 4th premise true, since $p = F$ we must require that $s = T$. However, this combination makes the 3rd premise false. Thus we move onto the second case.

In the second case, we still must require that $s = T$ since $p = F$ still. With $s = T$ and $r = T$, the third premise is true. It also makes the second and first premises true. Therefore, $s = T$, $p = F$, $q = F$ and $r = T$ is a combination which yields true premises and a false conclusion. The argument is invalid.