Math 2315 - Calculus II

Final Exam - 2006.04.24

Due Date - 2006.05.10 by 11:00 AM

Name: ____________________________

Please answer each question as fully as possible. Partial credit will only be given if you show your work. Also, make sure your work is neat and orderly. Finally, DO NOT work together, DO NOT get assistance from anyone but me!

1. If the region $\mathfrak{R} = \{(x, y) \mid x \geq 1,\ 0 \leq y \leq \frac{1}{x}\}$ is rotated about the x-axis, show that the volume of the resulting solid is finite, but that its surface area is infinite.

2. Evaluate the following integrals:

a)

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2}dx$$

b)

$$\int x^8 e^x - x^4 \cos^2(x) dx$$

c)

$$\int \frac{dx}{\sqrt{25x^2-4}}, \quad x > \frac{2}{5}$$

d)

$$\int_1^\infty \frac{dx}{x\sqrt{x^2-1}}$$

3. The parametric equations for an *astroid* are:

$$\begin{cases} x(t) = \cos^3(t) \\ y(t) = \sin^3(t), \end{cases}$$

with $0 \leq t \leq 2\pi$. Calculate the arc length of 1/4 of the astroid ($0 \leq t \leq \pi/2$).

4. Consider the limaçon, defined by the polar equation $r = 2\cos(\theta) + 1$. Graph the limaçon in the xy - coordinate plane.

5. Find the area inside the smaller loop of the limaçon graphed in problem 4.

6. Determine if the following sequences converge or diverge. If convergent state the limit, if divergent state a reason why.

a)

$$a_n = \frac{\ln\left(n^2\right)}{n}$$

b)

$$a_n = \sinh(\ln(n))$$

c)

$$\frac{n^2}{2n-1}\sin\left(\frac{1}{n}\right)$$

7. Show by example that $\sum(a_n/b_n)$ may diverge even though $\sum a_n$ and $\sum b_n$ converge and no b_n equals 0.

8. Determine if the following series converge or diverge.

a)

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$$

b)

$$\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n^2+n} - n\right)$$

c)

$$\sum_{n=1}^{\infty} \frac{n^4}{2^n}$$

c)

$$\sum_{n=1}^{\infty} \frac{n!\ln(n)}{n(n+2)!}$$

9. Show that the Ratio Test and the Root Test fail to provide information on the convergence of the p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}.$$

10. Use the Taylor series generated by e^x at $x = a$ to show that

$$e^x = e^a \left[1 + (x-a) + \frac{(x-a)^2}{2!} + \cdots\right]$$

11. Find the first four terms (up to the x^3 term) in the Taylor series expansion of $f(x) = e^{\sin(x)}$. This is sometimes referred to as a third order approximation, similar to tangent line approximations.

12. How much of the assigned textbook homework did you actually do over the course of the semester? Please answer honestly, your grade will not be affected either way.