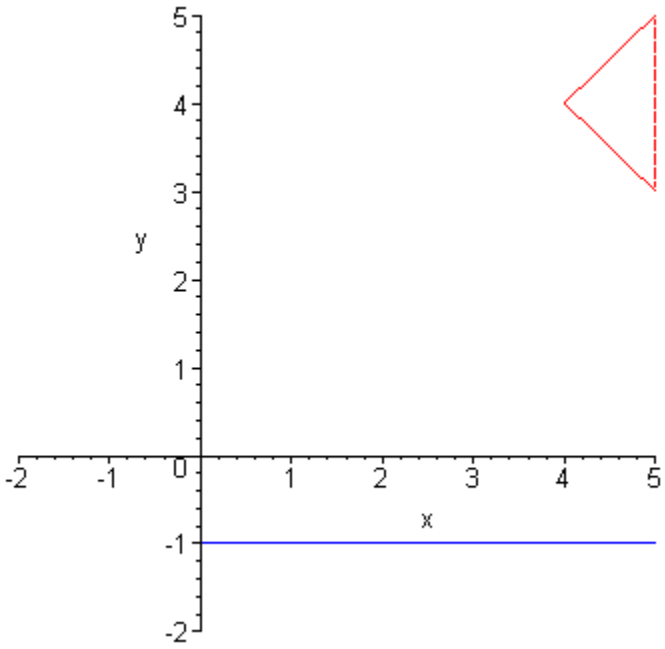
Math 2315 - Calculus II

Quiz #4 - 2006.02.10

Solutions

Find the volume of the surface bounded by the intersections of the curves $y = x$, $y = -x + 8$ and $x = 5$ which is rotated about $y = -1$. Use BOTH methods, you DO NOT have to compute the integrals, just set them up.



The volume is given can be computed by either method quite readily. Here we will start with the washer method. The outside radius is given by $y = x$, ad the inside by $y = -x + 8$. Therefore, we have

$$A = \int_4^5 \pi \left(x^2 - (-x+8)^2\right) dx.$$

By the method of cylindrical shells, we must break up the integral as follows:

$$A = \int_3^4 2\pi(y+1)(5-(-y+8))dy + \int_4^5 2\pi(y+1)(5-y)dy.$$

Notice that

$$A = \int_4^5 \pi \left(x^2 - (-x+8)^2\right) dx = \int_3^4 2\pi y(5-(-y+8))dy + \int_4^5 2\pi y(5-y)dy = 8\pi.$$