Math 3113 - Multivariable Calculus

Homework #4 - 2006.02.20

Due Date - 2006.02.27

Solutions

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1. Describe (do not draw) the graphs of the following spherical equations.

a) $\rho = 2\cos(\phi)$

Multiplying both sides by ρ gives $\rho^2 = 2\rho\cos(\phi)$. This of course is now given in standard coordinates by $x^2 + y^2 + z^2 = 2z$. Completing the square gives $x^2 + y^2 + (z-1)^2 = 1$. So the graph is a sphere with center $(0, 0, 1)$ and radius 1.

b) $\rho = \sin(\phi)\sin(\theta)$

As in a), multiplying both sides of the equation by ρ gives $\rho^2 = \rho\sin(\phi)\sin(\theta)$. Similarly, this yields $x^2+y^2+z^2 = y$. After completing the square, one sees that this is a sphere with radius $\frac{1}{2}$ centered at $\left(0, \frac{1}{2}, 0\right)$.

2. Describe (do not draw) the graphs of the following cylindrical equations.

a) $\cos(\theta) + \sin(\theta) = 0$

This can be written as $\frac{1}{r}(x + y) = 0$, or $y = -x$. This is of course a vertical plane which traces the line $y = -x$ in the xy plane.

b) $z = 10 - 3r^2$

This is a paraboloid opening downward at $z = 10$.

3. Consider the equation for the paraboloid $z = x^2 + y^2$.

a) Find a cylindrical coordinate equation for the paraboloid given above.

This is just plainly $z = r^2$.

b) Find a spherical coordinate equation for the paraboloid given above.

Here, substitute $z = \rho\cos(\phi)$ and $x^2 + y^2 + z^2 = \rho^2$ to get $\rho\cos(\phi) = \rho^2\sin^2(\phi)$. If one cancels a ρ, it can be simplified to $\rho = \csc(\phi)\cot(\phi)$.