Math 3113 - Multivariable Calculus

Homework #5 - 2006.03.07
Due Date - 2006.03.10
Solutions

1. Determine the arc length function $s(t)$ for the function $\overrightarrow{r(t)} = \langle 2t, 3\sin(2t), 3\cos(2t)\rangle$ starting at time $t = 0$.

$$s(t) = \int_0^t \sqrt{4 + 36\cos^2(2s) + 36\sin^2(2s)}ds = 2\sqrt{10}t$$

2. Reparameterize the function $\overrightarrow{r(t)}$ in terms of the arc length function.
Since $t = \frac{s}{2\sqrt{10}}$, we have

$$\overrightarrow{r(t(s))} = \langle \frac{s}{\sqrt{10}}, 3\sin\left(\frac{s}{\sqrt{10}}\right), 3\cos\left(\frac{s}{\sqrt{10}}\right)\rangle$$

3. Where on the curve $\overrightarrow{r(t)}$ are we after traveling for a distance of $\frac{\sqrt{10}}{3}\pi$?
Substituting $s = \frac{\sqrt{10}}{3}\pi$ into $\overrightarrow{r(t(s))}$ gives

$$\overrightarrow{r\left(t\left(\frac{\sqrt{10}}{3}\pi\right)\right)} = \langle \frac{\pi}{3}, \frac{3\sqrt{3}}{2}, \frac{3}{2}\rangle$$

4. Find the equation of the unit tangent vector to the curve $\overrightarrow{r(t)}$ at $t = \pi$.

$$r'(t) = \langle 2, 6\cos(2t), -6\sin(2t)\rangle$$

and

$$r'(\pi) = \langle 2, 6, 0\rangle$$

with

$$|r'(\pi)| = 2\sqrt{10}$$

and

$$T(\pi) = \langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}, 0\rangle$$

5. Write down a the equation for a space curve $\overrightarrow{R(t)}$ such that the curve intersects itself at some point in space (at different times obviously) with different tangent vectors.
Answers will vary.

6. Write down the unit tangent vectors for $\overrightarrow{R(t)}$ at the times in question above at the point of intersection.
Answers will vary.