

Math 3113 - Multivariable Calculus

Homework #6 - 2006.03.23

Due Date - 2006.03.29

Name: _____

1. Sketch a contour map of each of the following functions.

a) $f(x, y) = 1 - |x| - |y|$

b) $f(x, y) = 4x^2 + y^2$

c) $f(x, y) = x^2 - 4y^2$

d) $f(x, y) = \ln\left(\sqrt{x^2 + y^2} + 1\right)$

2. Consider the function

$$f(x, y) = \frac{x^2(x + y)}{x^4 + (x + y)^2}.$$

a) Show that $f(x, y) \rightarrow 0$ as $(x, y) \rightarrow (0, 0)$ for all curves of the form $y = ax^k$ for $k \geq 1$.

b) Show that $f(x, y) \rightarrow 0$ as $(x, y) \rightarrow (0, 0)$ for all curves of the form $x = ay^k$ for $k \geq 1$.

c) Show that $f(x, y) \rightarrow \frac{1}{2}$ as $(x, y) \rightarrow (0, 0)$ along the curve $y = x^2 - x$.

3. Notice that the following two expressions are equivalent:

$$\lim_{(x,y) \rightarrow (a,b)} f(x, y) = \lim_{(x,y) \rightarrow (0,0)} f(x + a, y + b)$$

Since it is easier to work with curves through the origin, in limits where $(a, b) \neq (0, 0)$ and the limit is not obvious, one can replace x by $x + a$ and y by $y + b$ and let $(x, y) \rightarrow (0, 0)$.

Consider the following limit:

$$\lim_{(x,y) \rightarrow (1,0)} \frac{4(x - 1)^4(y + 1) + 4y^2}{(x - 1)^4 + y^2}.$$

a) Rewrite the above limit so that the limit goes through the origin instead of through the point $(1, 0)$.

b) Using the STRICT definition of the limit, show that the limit in part a) is equal to 4.

c) What value does δ have to be smaller than if $\varepsilon = \frac{1}{25}$?