Math 3113 - Multivariable Calculus

Homework #7 - 2006.03.29

Due Date - 2006.04.03

Name: ______________________________

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1. Consider the three dimensional Laplace equation given by

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$

Which of the following functions satisfy the Laplace equation?
a) $f(x, y, z) = x^2 + y^2 - 2z^2$
b) $f(x, y, z) = 2z^3 - 3\left(x^2 + y^2\right) z$
c) $f(x, y, z) = e^{-2y} \cos(2x) + z$
d) $f(x, y, z) = \left(x^2 + y^2 + z^2\right)^{-\frac{1}{2}}$
e) $f(x, y, z) = e^{3x+4y} \cos(5z)$.

2. Consider the one dimensional wave equation given by

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2},$$

where w is the wave height, x is the distance variable, t is time and c is the velocity of the wave being propagated. Which of the following functions satisfy the wave equation?
a) $w(x, t) = \sin(x + ct)$
b) $w(x, t) = 5 \cos(3x + 3ct) + e^{x+ct}$
c) $w(x, t) = \sin(x + ct) + \cos(2x + 2ct)$
d) $w(x, t) = \ln(2x + 2ct)$
e) $w(x, t) = \tan(2x - 2ct)$

3. Show that the if $f(z)$ is a differentiable function, then $u = f(z(x, t))$ with $z = x - t$, will always satisfy the partial differential equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 0,$$

independent of the actual form of f.

4. Show that the if $f(z)$ is a differentiable function, then $w = f(z(x, t))$ where $z = x + ct$, will always satisfy the one dimensional wave equation given in problem 2.