Math 3113 - Multivariable Calculus

Homework #8 - 2006.04.05

Due Date - 2006.04.12

Name: ______________________________

A function $f(x, y)$ is said to be homogeneous of degree n if $f(tx, ty) = t^n f(x, y)$ for all t. Here n is a positive integer. Homogeneous functions are very important in the study of elliptic curves and cryptography.

1. Show that the function $r(x, y) = 4xy^6 - 2x^3y^4 + x^7$ is homogeneous of degree 7.

2. Give a nontrivial example of a function $g(x, y)$ which is homogeneous of degree 9.

3. Show that if $f(x, y)$ is homogeneous of degree n and sufficiently differentiable, then $f(x, y)$ satisfies the equation

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y).$$

(Hint: let $X = tx$ and $Y = ty$ and take the derivative with respect to t of the equation $f(tx, ty) = t^n f(x, y)$ and consider the case of $t = 1$.)

4. Using the assumptions in problem 3, show that $f(x, y)$ also satisfies

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f(x, y).$$