Math 3113 - Multivariable Calculus

Homework #9 - 2006.04.13

Due Date - 2006.04.21

Solutions

1. There are points on the surface $(y+z)^2 + (z-x)^2 = 16$ where the normal line is parallel to the yz-plane. These points lie on 2 lines, express these two lines in parametric form.

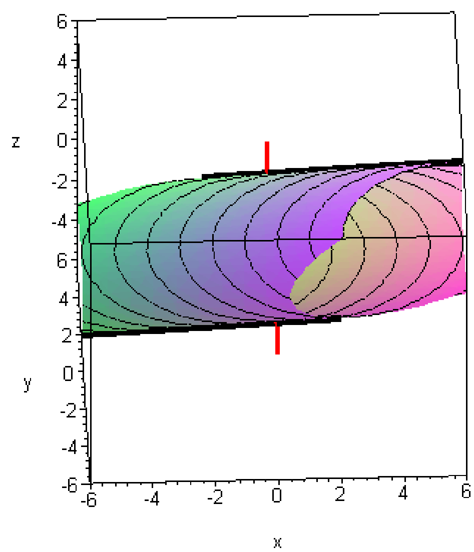
The normal line is given by the gradient vector. For the gradient vector to be parallel to the yz-plane, the x component must be zero. So if $f(x,y,z) = (y+z)^2 + (z-x)^2$, then we are considering the level surface $f(x,y,z) = 16$. Also note that

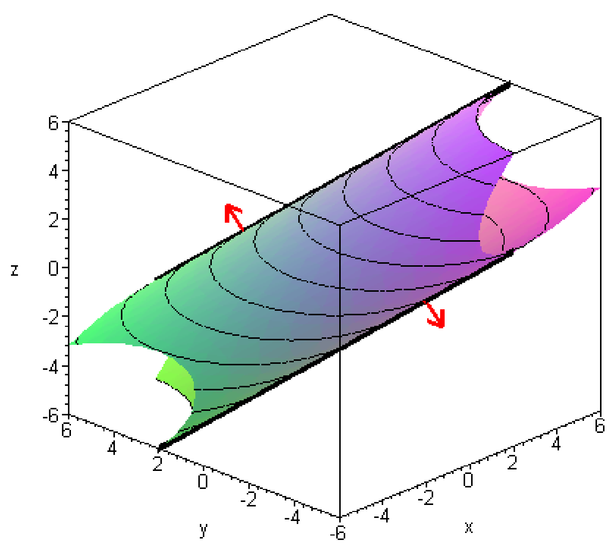
$$\nabla f = \langle 2x - 2z, 2y + 2z, 4z + 2y - 2x \rangle,$$

and setting the $2x - 2z = 0$ gives $x = z$. If $x = z$, then $f(x,y,x) = 16$ implies that $y = -x \pm 4$. Therefore, the parametric equations of the lines in question are

$$\begin{cases} x = t \\ y = -t \pm 4 \\ z = t. \end{cases}$$

The figures depict the surface, the two parametric lines defined above (black) and a normal vector (red) at a point on each of the lines. In the left figure, notice that it is clear that the normal vector lies in a plane parallel to the yz-plane. The right picture gives a better view of the surface and the parametric lines.





2. Find the two points on the surface $xy + yz + zx - x - z^2 = 0$ where the tangent plane is parallel to the xy-plane.

For the tangent plane to be parallel to the xy-plane, the normal direction must point in the z direction only. Thus if $g(x, y, z) = xy + yz + zx - x - z^2$, we are looking a the level surface $g(x, y, z) = 0$. Next, we compute the normal direction

$$\nabla g = \langle y + z - 1, x + z, y - 2z \rangle,$$

which we want to be parallel to $\hat{k}$ which requires

$$y + z - 1 = 0, \quad x + z = 0.$$

This means that the points in question lie on the parametric line defined by

$$\begin{cases} x = t \\ y = 1 + t \\ z = -t. \end{cases}$$

Substitution this back into g, we have $g(t, 1 + t, -t) = -2t^2 - t = 0$. Thus $t = 0$ and $t = -\frac{1}{2}$ are the times on the parametric line which intersect the level surface g. The points in question are thus given by $(0, 1, 0)$ and $(-1/2, 1/2, 1/2)$.

The following figure shows the surface and its contours along with the parametric line defined above (in black) and the two points on the line which intersect the surface (red).