

Math 3113 - Multivariable Calculus

Quiz #11 - 2006.04.05

Solutions

Consider the function $f(r, \theta) = \sqrt{r^2 + \theta^3} \sin^2(2\theta - 3r)$. Answer the following questions.

1. Compute $\frac{\partial f}{\partial \theta}$.

$$\frac{\partial f}{\partial \theta} = \frac{3}{2} \frac{\theta^2 \sin^2(2\theta - 3r)}{\sqrt{r^2 + \theta^3}} + 4\sqrt{r^2 + \theta^3} \sin(2\theta - 3r) \cos(2\theta - 3r)$$

2. Compute $\frac{\partial f}{\partial r}$.

$$\frac{\partial f}{\partial r} = \frac{r \sin^2(2\theta - 3r)}{\sqrt{r^2 + \theta^3}} - 6\sqrt{r^2 + \theta^3} \sin(2\theta - 3r) \cos(2\theta - 3r)$$

3. Compute $\frac{\partial^2 f}{\partial r \partial \theta}$.

$$\begin{aligned} \frac{\partial^2 f}{\partial r \partial \theta} = & -\frac{3}{2} \frac{r\theta^2 \sin^2(2\theta - 3r)}{(r^2 + \theta^3)^{\frac{3}{2}}} + \frac{(4r - 9\theta^2) \sin(2\theta - 3r) \cos(2\theta - 3r)}{\sqrt{r^2 + \theta^3}} \\ & + 12\sqrt{r^2 + \theta^3} (\sin^2(2\theta - 3r) - \cos^2(2\theta - 3r)) \end{aligned}$$

4. Compute $\frac{\partial^2 f}{\partial \theta \partial r}$.

This answer is the same as the answer to problem 3.