Math 3113 - Multivariable Calculus

Quiz #12 - 2006.04.07

Solutions

Consider the function $R = \sin(x+y)\cos(x-y)$, with $x(s,t) = \frac{s}{t}$, and $y(s,t) = st$. Answer the following questions.

1. Compute $\frac{\partial R}{\partial t}$

First we notice that

$$\frac{\partial R}{\partial t} = \frac{\partial R}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial t}.$$

With

$$\frac{\partial R}{\partial x} = \cos(x+y)\cos(x-y) - \sin(x+y)\sin(x-y),$$
$$\frac{\partial R}{\partial y} = \cos(x+y)\cos(x-y) + \sin(x+y)\sin(x-y),$$
$$\frac{\partial x}{\partial t} = -\frac{s}{t^2}$$

and

$$\frac{\partial y}{\partial t} = s.$$

Putting all of this together gives

$$\frac{\partial R}{\partial t} = (\cos(x+y)\cos(x-y) - \sin(x+y)\sin(x-y))\left(-\frac{s}{t^2}\right) + (\cos(x+y)\cos(x-y) + \sin(x+y)\sin(x-y))\,s,$$

where x and y are defined as in the problem.

2. Compute $\frac{\partial R}{\partial s}$

$$\frac{\partial R}{\partial s} = \frac{\partial R}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial s}.$$

The only new information we need here is:

$$\frac{\partial x}{\partial s} = \frac{1}{t}$$

and

$$\frac{\partial y}{\partial s} = t.$$

$$\frac{\partial R}{\partial s} = (\cos(x+y)\cos(x-y) - \sin(x+y)\sin(x-y))\left(\frac{1}{t}\right) + (\cos(x+y)\cos(x-y) + \sin(x+y)\sin(x-y))\,t,$$