

Math 3113 - Multivariable Calculus

Quiz #13 - 2006.04.12

Solutions

For the following problems, let $\vec{v} = \langle 2, -2 \rangle$, $P = (-1, 1)$ and $f(x, y) = e^{x+y} (x^2 - y^2)$.

1. Find a unit vector \vec{u} in the direction of \vec{v} .

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 2, -2 \rangle}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \langle 1, -1 \rangle$$

2. Compute ∇f .

$$\nabla f = \langle f_x, f_y \rangle = \langle e^{x+y} (x^2 - y^2 + 2x), e^{x+y} (x^2 - y^2 - 2y) \rangle$$

3. Find the directional derivative of $f(x, y)$ at the point P in the direction of the vector \vec{v} .

First, note that

$$\nabla f(-1, 1) = \langle -2, -2 \rangle$$

and thus

$$\nabla f(-1, 1) \cdot \vec{u} = \frac{1}{\sqrt{2}} \langle 1, -1 \rangle \cdot \langle -2, -2 \rangle = 0.$$

4. In what direction does the directional derivative obtain its maximum at the point P ?

The directional derivative always obtains its maximum in the direction of the gradient, this at P , the directional derivative is a maximum in the direction $\langle -2, -2 \rangle$.

5. What is the value of the directional derivative in the direction found in problem 4 at point P .

The value is simply $|\nabla f| = 2\sqrt{2}$.