Math 3113 - Multivariable Calculus

Quiz #7 - 2006.03.08

Solutions

Consider the curve $\overrightarrow{r(t)} = \langle t, \sin\left(\frac{t}{2}\right), \cos(t)\rangle$.

1. Compute the unit tangent vector $T(t)$.

$$T(t) = \frac{\langle 1, \frac{1}{2}\cos\left(\frac{t}{2}\right), -\sin(t)\rangle}{\sqrt{1 + \frac{1}{4}\cos^2\left(\frac{t}{2}\right) + \sin^2(t)}}$$

2. Can $T(t)$ ever be parallel to any of the three coordinate axes? Why or why not? If so, which axis or axes?

To be parallel to an axis, $T(t)$ must be a multiple of one of the unit vectors $\hat{i}$, $\hat{j}$ or $\hat{k}$. Notice that $\hat{i}$ component of $T(t)$ is never zero. Therefore, the only possible axis to be parallel to is the x-axis. This would require that $\cos\left(\frac{t}{2}\right) = \sin(t) = 0$. This occurs when t is an odd multiple of π.

3. Can $T(t)$ ever lie in one of the three (xy, xz and yz) coordinate planes ? Why or why not? If so, which ones?

Here, all that is required is that one of the components of $T(t)$ be zero. This can obviously only happen in the $\hat{j}$ and $\hat{k}$ components. Thus, $T(t)$ can lie in planes parallel to the xz and xy planes.