

Math 3113 - Multivariable Calculus

Quiz #8 - 2006.03.10

Solutions

Consider an object traveling along the curve $\vec{r}(t) = \langle \cos(t) + t \sin(t), \sin(t) - t \cos(t), 1 \rangle$ for $t \geq 0$. Answer the following questions concerning $\vec{r}(t)$.

1. Compute the arc length function $s(t)$ for $\vec{r}(t)$ defined as above.

First, we compute $r'(t)$.

$$r'(t) = \langle t \cos(t), t \sin(t), 0 \rangle.$$

The by definition,

$$s(t) = \int_0^t |r'(u)| du = \int_0^t \sqrt{u^2 \cos^2(u) + u^2 \sin^2(u)} du = \int_0^t u du = \frac{1}{2}t^2.$$

2. How long does it take the object to travel a distance of $2\pi^2$ units from its starting point at time $t = 0$?

Notice that from problem 1, $t = \sqrt{2s}$. Therefore, if $s = 2\pi^2$, then $t = 2\pi$. So it takes 2π time units to travel a distance of $2\pi^2$ starting at time $t = 0$.

3. Where is the object located in (x, y, z) coordinates after it has traveled a distance of 10 units?

When $t = \sqrt{20}$, $r(\sqrt{20}) = (\cos(\sqrt{20}) + \sqrt{20} \sin(\sqrt{20}), \sin(\sqrt{20}) - \sqrt{20} \cos(\sqrt{20}), 1)$