Math 3213 - Differential Equations

Quiz #3 - 2007.02.02

Solutions

Find the general solution to the following differential equations:

1.

$$y' = \frac{xy - y}{y + 1}$$

This equation is separable:

$$\left(\frac{y+1}{y}\right)\frac{dy}{=}(x-1)dx.$$

We simplify the left side first:

$$\left(1 + \frac{1}{y}\right)\frac{dy}{=}(x-1)dx.$$

Integrating gives

$$y + \ln(|y|) = \frac{1}{2}x^2 - x + c.$$

2.

$$y' - 2xy = x$$

This is a first order linear equation. As such, we have $\mu(x) = e^{-x^2}$. Thus

$$\frac{d}{dx}\left(e^{-x^2}y\right) = e^{-x^2}x.$$

Integrating both sides gives

$$e^{-x^2}y = -\frac{1}{2}e^{-x^2} + c.$$

Solving for y:

$$y(x) = -\frac{1}{2} + ce^{x^2}.$$