Math 3213 - Differential Equations

Quiz #8 - 2007.03.05

Solutions

Solve the following second order differential equation:

$$y'' + y' - 6y = 2\cos(3x)$$

The characteristic equation is given by $r^2 + r - 6 = 0$ which has roots $r_1 = 2$ and $r_2 = -3$. So the solution to the homogeneous equation is

$$y_h(x) = c_1 e^{2x} + c_2 e^{-3x}.$$

Now we find the particular solution. So we guess

$$y_p(x) = A\cos(3x) + B\sin(3x).$$

Thus we have

$$y_p'(x) = -3A\sin(3x) + 3B\cos(3x)$$
$$y_p''(x) = -9A\cos(3x) - 9B\sin(3x),$$

and putting this together gives the system of equations for the coefficients A and B to be

$$\begin{cases} -15A + 3B &= 2 \\ -3A - 15B &= 0. \end{cases}$$

So we have $B = \frac{1}{39}$ and $A = -\frac{5}{39}$, and our final solution is

$$y(x) = c_1 e^{2x} + c_2 e^{-3x} - \frac{5}{39}\cos(3x) + \frac{1}{39}\sin(3x).$$