Physics 1114 - General Physics I

Midterm

Solutions

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1. Your cat drops from a shelf 1.22 m above the floor and lands on all four feet. His legs bring him to a stop in a distance of 12 cm. Ignoring air resistance, calculate his speed when he first touches down on the floor. Furthermore, what is his acceleration while he stopped himself over those 12 cm?

Here we will assume that the positive direction of motion is upward. As a result, $a = -g$, $v_{0y} = 0$, $\Delta y = -1.22$m. We use

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y \tag{1}$$

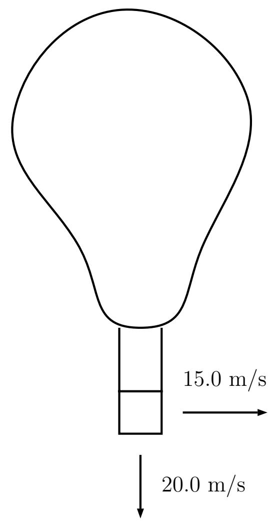
and solve for v_y:

$$\begin{aligned} v_y &= \sqrt{v_{0y}^2 + 2a_y \Delta y} \\ &= \sqrt{-2g \cdot (-1.22)} \\ &= -4.9 \text{ m/s} \end{aligned}$$

To find his acceleration after hitting the floor, note that we have an initial velocity of $v_{0y} = -4.9$m/s from previous, and $\Delta y = -0.12$ m. Since the cat comes to a stop, $v_y = 0$. We can once again use equation (1), this time solving for a_y:

$$\begin{aligned} a_y &= \frac{v_y^2 - v_{0y}^2}{2\Delta y} \\ &= \frac{0^2 - (-4.89)^2}{2(-0.12)} \\ &= 99.6 \text{ m/s}^2 \end{aligned}$$

2. A balloon carrying a basket is descending at a constant velocity of 20.0 m/s. A person in the basket throws a stone with an initial velocity of 15.0 m/s horizontally perpendicular to the path of the descending balloon, and 4.00 s later, this person sees the rock strike the ground. (a) How high was the balloon when the rock was thrown? (b) How far horizontally does the rock travel before it hits the ground? (c) at the instant the rock hits the ground, how far is it from the basket?



So we have two directions of motion here. We will assume the origin at the point where the stone is thrown. Thus, the stone's acceleration is $a_x = 0$ m/s^2 and $a_y = -9.8$ m/s^2. The stone's initial velocity is $v_{0x} = 15.0$ m/s and $v_{0y} = 20.0$ m/s.

(a) To find the vertical distance traveled, we use

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 \tag{2}$$

This gives

$$\begin{aligned}\Delta y &= (20.0 \text{ m/s})(4.00 \text{ s}) + \frac{1}{2}(9.80 \text{ m/s}^2)(4.00 \text{ s})^2 \\ &= 158 \text{ m}\end{aligned}$$

(b) To find the horizontal distance we use

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2 \tag{3}$$

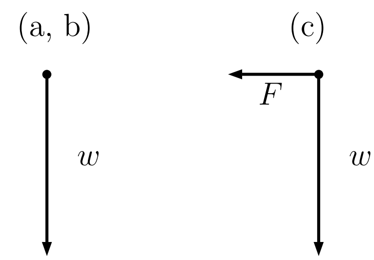
and remember, $a_x = 0$ for projectile motion, thus

$$\begin{aligned}\Delta x &= v_{0x}t \\ &= (15.0 \text{ m/s})(4.00 \text{ s}) = 60.0 \text{ m}\end{aligned}$$

(c) To find the distance between the stone and the basket, note that the balloon travels under constant velocity downward for four seconds between the time the stone was thrown and when it landed. This gives a vertical change in distance of $(20.0 \text{ m/s})(4.00 \text{ s}) = 80.0$ m. Since the balloon was 158.0 m above the ground to start with, it is now 78.0 m above the ground. Furthermore, the stone was displaces 60.0 m horizontally from the balloon so we use the Pythagorean formula to get

$$D = \sqrt{x^2 + y^2} = \sqrt{60^2 + 78^2} \text{ m} = 98.4 \text{ m}$$

3. A tennis ball travelling horizontally at 22 m/s suddenly hits a vertical brick wall and bounces back with horizontal velocity of 18 m/s. Make a free-body diagram of this ball (a) just before it hits the wall, (b) just after it has bounced free of the wall, and (c) while it is in contact with the wall.



For parts (a) and (b), the only force acting on the tennis ball is its weight (mass times gravity). For part (c), the wall is exerting a force against the direction of motion. In the figure above, we assume the ball is was traveling to the right when it hit the wall.

4. An unstretched spring is 12.00 cm long. When you hang a 0.876 kg weight from it, it stretches to a length of 14.40 cm. (a) What is the force constant (in N/m) of this spring? (b) What total mass must you hang from the spring to stretch it to a total length of 17.72 cm?

(a) The force exerted on the spring by the weight is $F = mg/$ However, from Hooke's Law, this must also be $F = kx$, setting these two equal gives $mg = kx$. We need k and have m, g and x:

$$k = \frac{mg}{x} = \frac{(0.875 \text{ kg})(9.80 \text{ m/s}^2)}{0.0240 \text{ m}} = 357\text{N/m}$$

For part (b), we use the same formula $mg = kx$ but solve for m instead, using the value of k from part (a):

$$m = \frac{kx}{g} = \frac{(357\text{N/m})(0.0572 \text{ m})}{9.80 \text{ m/s}^2} = 2.08 \text{ kg}$$

5. If two tiny identical spheres attract each other with a force of 3.0 nN when they are 25 cm apart, what is the mass of each sphere?

We will use the formula

$$F_g = G\frac{m_1 m_2}{r^2} \tag{4}$$

and note that $m_1 = m_2 = m$ is the unknown value we need to find. Solving for m gives

$$m = r\sqrt{\frac{F_g}{G}}$$

Using $F_g = 3.0 \times 10^{-9}$ N and $G = 6.673 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ gives

$$m = 1.7\ \mathrm{kg}$$

6. The mass of the moon is about 1/81 the mass of the earth, its radius is 1/4 that of the earth, and the acceleration due to gravity at the earth's surface is 9.80 m/s^2. Without looking up either body's mass, use this information to compute the acceleration due to gravity on the moon's surface.

For an object of mass m, the force due to gravity on the earth is given by

$$F_{g,e} = G\frac{m\, m_e}{r_e^2} \tag{5}$$

where m_e and r_e are the mass and radius of the earth, respectively. Similarly for the moonm

$$F_{g,m} = G\frac{m\, m_m}{r_m^2} \tag{6}$$

Note that in each of these, $F_{g,e} = m\, g_e$ and $F_{g,m} = m\, g_m$. Setting the right sides of equations (5) and (6) to $m\, g_e$ and $m\, g_m$ and dividing by m gives

$$g_e = G\frac{m_e}{r_e^2} = 9.80 \text{ m/s}^2, \tag{7}$$

$$g_m = G\frac{m_m}{r_m^2} \tag{8}$$

We really want g_m, and we can use the fact that $m_m = \frac{1}{81}m_e$ and $r_m = \frac{1}{4}r_e$ to plug into equation (8) to get:

$$g_m = G\frac{1/81 m_e}{(1/4 r_e)^2} = \frac{16}{81}\, G\, \frac{m_e}{r_e^2} = \frac{16}{81} g_e \approx 1.9 \text{ m/s}^2$$

7. You observe an object in free fall. When you first notice it, you determine that it is traveling at 15 m/s. A short time later, the object is traveling at 30 m/s. How much has gravitational potential energy changed during this time? (Here you may assume that wind resistance is negligible and that gravity is the only force acting on the stone in free fall.)

Here we simply use conservation of mechanical energy. Call initial time t_0 and final time t_f. At time t_0, $E_0 = mgy_0 + \frac{1}{2}mv_0^2$. Similarly, at time t_f, we have $E_f = mgy_f + \frac{1}{2}mv_f^2$. Setting these two equal gives

$$mgy_0 + \frac{1}{2}mv_0^2 = mgy_f + \frac{1}{2}mv_f^2$$

Moving gy_f to the left side and $\frac{1}{2}v_0^2$ to the right gives:

$$mg(y_0 - y_f) = \frac{1}{2}m\left(v_f^2 - v_0^2\right)$$

Setting $h = y_0 - y_f$ gives

$$mgh = \frac{1}{2}m\left(v_f^2 - v_0^2\right)$$

The left side is gravitational potential. The right side is $337.5 \cdot m$ J, where m is the unknown mass of the object in kg.

8. An unstretched spring has a force constant of 1200 N/m. How large a force and how much work are required to stretch the spring: (a) by 1.0 m from its unstretched length, (b) by 2.0 m, and (c) how much additional work did it take stretch the spring from 1.0 m to 2.0 m?

Remember that $F = kx$ and the $W = \frac{1}{2}kx^2$.

For part (a), $F_{1.0} = (1200 \text{ N/m})(1.0 \text{ m}) = 1200 \text{ N}$ and $W_{1.0} = \frac{1}{2}(1200 \text{ N/m})(1.0 \text{ m})^2 = 600 \text{ J}$.

For part (b), $F_{2.0} = (1200 \text{ N/m})(2.0 \text{ m}) = 2400 \text{ N}$ and $W_{2.0} = \frac{1}{2}(1200 \text{ N/m})(2.0 \text{ m})^2 = 2400 \text{ J}$.

Finally for part (c) we have $\Delta W = W_{2.0} - W_{1.0} = 1800 \text{ J}$.

9. A rifle bullet with mass 8.00 g strikes and embeds itself in a block with a mass of 0.992 kg that rests on a frictionless, horizontal surface and is attached to a coil spring. The impact compresses the spring 15.0 cm. Calibration of the spring shows that a force of 0.750 N is required to compress the spring 0.250 cm. (a) Find the magnitude of the block's velocity just after impact. (b) What was the initial speed of the bullet?

We will apply both conservation of momentum and conservation of energy here. But first, we must compute the spring constant from the given information using $F = kx$, solving for k:

$$k = \frac{F}{x} = \frac{0.750 \text{ N}}{0.250 \times 10^{-2} \text{ m}} = 300 \text{ N/m}$$

We next consider the spring-bullet system. For part (a), note that right at impact, the only moving object is the block+bullet weighing 1.00 kg. The spring is not compressed. So $E_i = 1/2\, mV^2$, where m is the mass of the bullet and block together, and V is the velocity of the block+bullet in it right after impact. When the spring is fully compressed, the block and bullet are no longer moving, and thus final energy is purely spring potential: $E_f = 1/2\, kx^2$.

We set these two equal to get

$$\frac{1}{2}mV^2 = \frac{1}{2}kx^2$$

The only unknown is V, which we can solve for:

$$\begin{aligned} V &= x\sqrt{\frac{k}{m}} \\ &= (0.150 \text{ m}\sqrt{\frac{300 \text{ N/m}}{1.00 \text{ kg}}} \\ &= 2.60 \text{ m/s} \end{aligned}$$

For part (b), we use conservation of momentum. We now know the final velocity of the block+bullet. Initially, the block is not moving and only the bullet is. Thus we have

$$m_{\text{bullet}} v_{\text{bullet}} = m_{\text{both}} v_{\text{both}}$$

We can solve for v_{bullet} to get

$$\begin{aligned} v_{\text{bullet}} &= \frac{m_{\text{both}} v_{\text{both}}}{m_{\text{bullet}}} \\ &= \frac{(1.00 \text{ kg})(2.60 \text{ m/s})}{8 \times 10^{-3} \text{ kg}} = 325 \text{ m/s} \end{aligned}$$