

# Math 1303 - Math in the Liberal Arts

Exam #1 - 2014.09.08

## Solutions

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1. Write the following statements in symbolic logic form. Be sure to define your simple statements first.

We will define our statements as follows:

$p$ : Thanksgiving is on Thursday,

$q$ : Christmas is on Saturday,

$r$ : New Years is on Monday.

- (a) Thanksgiving is on Thursday, and Christmas is on Saturday or New Years is on Monday.

$$p \wedge (q \vee r)$$

- (b) Thanksgiving is on Thursday and Christmas is on Saturday, or New Years is on Monday.

$$(p \wedge q) \vee r$$

- (c) If Thanksgiving is on Thursday then Christmas is on Saturday, or New Years is on Monday.

$$(p \rightarrow q) \vee r$$

- (d) If Thanksgiving is on Thursday, then Christmas is on Saturday or New Years is on Monday.

$$p \rightarrow (q \vee r)$$

2. Negate the following sentence: Today is Wednesday or we are not in logic class.

The form of this sentence is  $p \vee \sim q$ , so the negation will be  $\sim p \wedge q$ : Today is not Wednesday and we are in logic class.

3. Consider the following four statements:

$$p \vee (q \wedge r), \quad p \wedge (q \vee r), \quad (p \vee q) \wedge (p \vee r), \quad (p \wedge q) \vee (p \wedge r)$$

These four statements can be grouped into two pairs of equivalent statements, find them.

$p$	$q$	$r$	$(q \wedge r)$	$p \vee (q \wedge r)$	$p$	$q$	$r$	$(q \vee r)$	$p \wedge (q \vee r)$
T	T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	F	T	T
T	F	T	F	T	T	F	T	T	T
T	F	F	F	T	T	F	F	F	F
F	T	T	T	T	F	T	T	T	F
F	T	F	F	F	F	T	F	T	F
F	F	T	F	F	F	F	T	T	F
F	F	F	F	F	F	F	F	F	F

$p$	$q$	$r$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$p$	$q$	$r$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F	T	F	T
T	F	T	T	T	T	T	F	T	F	T	T
T	F	F	T	T	T	T	F	F	F	F	F
F	T	T	T	T	T	F	T	T	F	F	F
F	T	F	T	F	F	F	T	F	F	F	F
F	F	T	F	T	F	F	F	T	F	F	F
F	F	F	F	T	F	F	F	F	F	F	F

From the above truth tables, we see that  $p \vee (q \wedge r)$  and  $(p \vee q) \wedge (p \vee r)$  are equivalent and  $p \wedge (q \vee r)$  and  $(p \wedge q) \vee (p \wedge r)$  are equivalent.

4. Determine if the following argument is valid.

$$\begin{array}{l} p \vee q \\ p \rightarrow r \\ \hline \therefore q \vee r \end{array}$$

As a symbolic statement, the above argument is given by:  $[(p \vee q) \wedge (p \rightarrow r)] \rightarrow (q \vee r)$ . So we now construct a truth table for the argument.

$p$	$q$	$r$	$p \vee q$	$p \rightarrow r$	$(p \vee q) \wedge (p \rightarrow r)$	$q \vee r$	$[(p \vee q) \wedge (p \rightarrow r)] \rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

Since the final column is all T's, the argument is indeed valid!