

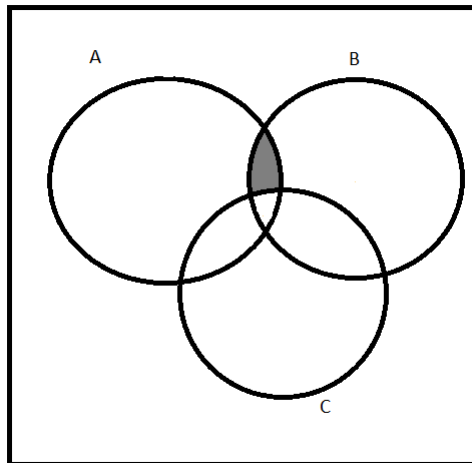
# Math 1303 - Math in the Liberal Arts

Exam #2 - 2014.10.06

Solutions

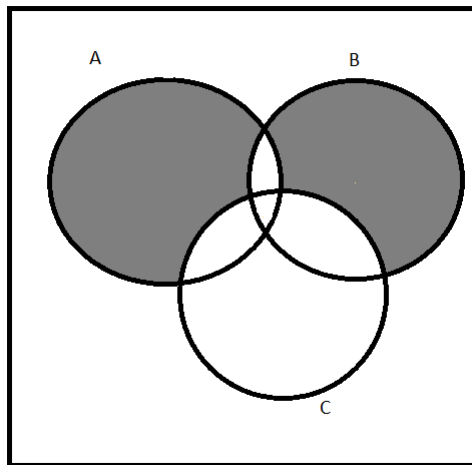
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1. Express the shaded region in the Venn diagram below in terms of only intersections of the sets  $A$ ,  $B$ ,  $C$ , and their complements.



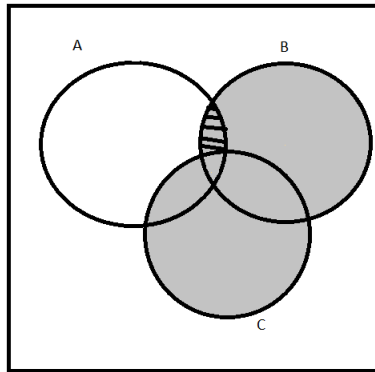
We can think of this as  $A$  and  $B$  and not  $C$ , thus the answer is  $A \cap B \cap C'$ .

2. Express the shaded region in the Venn diagram below in terms of unions and set differences of the sets  $A$ ,  $B$ ,  $C$ , and their complements.



We can think of this as two regions:  $A$  except anything in  $B$  or  $C$ , or  $B$  except anything in  $A$  or  $C$ . This gives  $(A - (B \cup C)) \cup (B - (A \cup C))$ .

3. Determine if the statement  $(A \cap B \cap C') \subset (B \cup C)$  is true by constructing a Venn diagram.



The lined region is  $(A \cap B \cap C')$  and the grey region is  $(B \cup C)$ . From this Venn diagram, it appears that the statement is true.

4. Determine if the statement  $(A \cap B \cap C') \subset (B \cup C)$  is true by converting the statement to logical form and constructing a truth table.

We first convert to a logical statement:

$$(x \in A \wedge x \in B \wedge \sim x \in C) \rightarrow (x \in B \vee x \in C)$$

Upon setting  $p = x \in A$ ,  $q = x \in B$  and  $r = x \in C$ , we have the statement

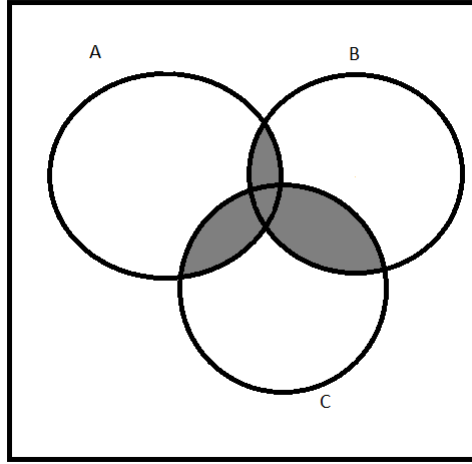
$$(p \wedge q \wedge \sim r) \rightarrow (q \vee r)$$

So now we do a truth table:

$p$	$q$	$r$	$p \wedge q$	$\sim r$	$p \wedge q \wedge \sim r$	$q \vee r$	$(p \wedge q \wedge \sim r) \rightarrow (q \vee r)$
T	T	T	T	F	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	T
F	T	T	F	F	F	T	T
F	T	F	F	T	F	T	T
F	F	T	F	F	F	T	T
F	F	F	F	T	F	F	T

The final column is all T's, so the statement is true.

5. For the following Venn diagram, determine which of the following sets corresponds to the shaded region. (There may be more than one correct answer).



- (a)  $(B \cap C) \cup (A \cap C) \cup (A \cap B)$  – Yes  
 (b)  $((B \cup C) - A) \cup (A \cap B \cap C)$  – No  
 (c)  $(A \cup B \cup C) - [(A - (B \cup C)) \cup ((B - (A \cup C)) \cup (C - (A \cup B)))]$  – Yes  
 (d)  $A' \cap B' \cap C'$  – No

6. Define the universe,  $U$ , to be all the set of all integers (both positive, negative and zero). Consider the following sets:

$$A = \{1, 2, 3, \dots, 25\}$$

$$B = \{-25, -24, -23, \dots, -1\}$$

$$C = \{1, 2, 3, \dots\}$$

$$D = \{\dots, -4, -2, 0, 2, 4, \dots\}$$

- (a) Compute  $D'$

$$D' = \{\dots, -3, -1, 1, 3, \dots\}$$

- (b) Compute  $A \cap B$

$$A \cap B = \{\}$$

- (c) Compute  $C'$

$$C' = \{\dots, -3, -2, -1, 0\}$$

- (d) Compute  $A \cap C$

$$A \cap C = A$$

- (e) Compute  $U - (A \cup B)$

$$U - (A \cup B) = \{\dots, -28, -27, -26, 0, 26, 27, 28, \dots\}$$

7. A group of students was polled about the courses they are enrolled in, and the answers were as follows:

There were no students took math but no gen ed or safety course.

14 students took all three types of courses, safety, math and gen ed.

8 students took none of the three types of courses.

23 students took gen ed and safety courses.

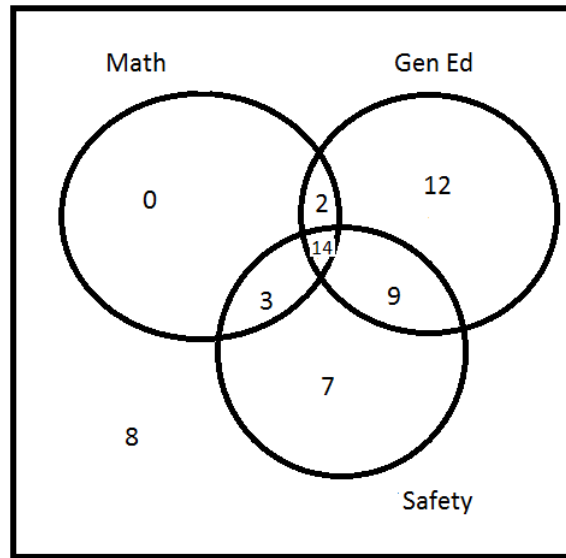
19 students took math classes.

33 students took safety courses.

37 students took gen ed courses.

There were 7 students who took safety courses but no math or gen ed courses.

Construct a Venn Diagram to illustrate the above sentences, and use it to determine how many students were polled.



Adding up all the areas, 55 students were polled.