

# Math 1303 - Math in the Liberal Arts

Quiz #10 - 2014.10.15

Solutions

---

Explain how the division by three rule works.

The division by three rule states that if the sum of the digits in the number is divisible by three, then the number is divisible by three. To see this, consider a number such as 5238. We can express this as

$$5238 = 5 \times 1000 + 2 \times 100 + 3 \times 10 + 8$$

But  $1000 = 999 + 1$ ,  $100 = 99 + 1$  and  $10 = 9 + 1$ . So we have

$$\begin{aligned} 5238 &= 5 \times (999 + 1) + 2 \times (99 + 1) + 3 \times (9 + 1) + 8 \\ &= 5 \times 999 + 2 \times 99 + 3 \times 9 + (5 + 2 + 3 + 9) \end{aligned}$$

Now, it is clear that 3 divides 999, 99 and 9. So all that remains is to ensure that 3 divides  $5 + 2 + 3 + 9$ . In general,  $10^k - 1$  is always divisible by 3, so what is left after expanding an arbitrary positive integer as in the previous example is the sum of the digits.