Math 1303 - Math in the Liberal Arts

Exam #3 - 2014.11.07

Solutions

For the following problems, let $x = 1980$, $y = 3150$, and $z = 924$.

1. Compute the prime factorization of the numbers x, y, and z.

$$x = 2^2 \cdot 3^2 \cdot 5 \cdot 11, \quad y = 2 \cdot 3^2 \cdot 5^2 \cdot 7, \quad z = 2^2 \cdot 3 \cdot 7 \cdot 11$$

2. Compute the GCD(x, y).

$$\text{GCD}(x, y) = 2 \cdot 3^2 \cdot 5 = 90$$

3. Compute the GCD(y, z).

$$\text{GCD}(y, z) = 2 \cdot 3 \cdot 7 = 42$$

4. Compute the GCD(GCD$(x, y), z)$.

$$\begin{aligned} \text{GCD}(\text{GCD}(x, y), z) &= \text{GCD}(90, 924) \\ &= 2 \cdot 3 = 6 \end{aligned}$$

5. Compute the GCD$(x, \text{GCD}(y, z))$.

$$\begin{aligned} \text{GCD}(x, \text{GCD}(y, z)) &= \text{GCD}(1980, 42) \\ &= 2 \cdot 3 = 6 \end{aligned}$$

6. Consider the set of all positive integers. Is the operation GCD closed with respect to the positive integers?

   Yes, GCD(x, y) will always give a positive integer, since at worst, GCD$(x, y) = 1$.

7. Is GCD commutative? I.e. is GCD$(x, y) = $ GCD(y, x)?

   Yes, comparing prime factorizations does not depend on order.

8. Is GCD associative? I.e. is GCD(GCD$(x, y), z) = $ GCD$(x, \text{GCD}(y, z))$?
   (See problems 4 and 5 for inspiration)

   Since we are comparing three prime factorizations, order does not matter.