Math 4133 - Linear Algebra

Quiz #10 - 2015.04.08

Solutions

For this quiz, let $\overrightarrow{u} = \langle -3, 7, 11 \rangle$ and $\overrightarrow{v} = \langle 6, -1, 9 \rangle$.

1. Find two vectors $\overrightarrow{w_1}$ and $\overrightarrow{w_2}$ perpendicular to $\overrightarrow{u}$ which are also perpendicular to each other.

First we find one vector, $\overrightarrow{w_1}$, which is perpendicular to $\overrightarrow{u}$. There are many such vectors, and we will simply require that $\overrightarrow{w_1} \cdot \overrightarrow{u} = 0$. Thus if $\overrightarrow{w_1} = \langle x, y, z \rangle$ we end up with the equation

$$-3x + 7y + 11z = 0$$

We have three unknowns and one equation which yields two degrees of freedom. If we let $x = 0$ and $y = 1$, we get $7 + 11z = 0$ or $z = -7/11$. So our first perpendicular vector could be $\overrightarrow{w_1} = \langle 0, 1, -7/11 \rangle$, but we will scale it to remove fractions:

$$\overrightarrow{w_1} = \langle 0, 11, -7 \rangle$$

To find $\overrightarrow{w_2}$, we simply take the cross product of $\overrightarrow{u}$ and $\overrightarrow{w_1}$:

$$\begin{aligned} \overrightarrow{w_2} = \overrightarrow{u} \times \overrightarrow{w_1} &= \det\left(\begin{bmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ -3 & 7 & 11 \\ 0 & 11 & -7 \end{bmatrix} \right) \\ &= -170\overrightarrow{i} - 21\overrightarrow{j} + (-33)\overrightarrow{k} \\ &= \langle -170, -12, -33 \rangle \end{aligned}$$

Obviously, answers will vary here, any two mutually perpendicular vectors which lie in the plane perpendicular to the vector $\overrightarrow{u}$ will suffice.

2. If you were to project either of the vectors you found from problem 1 onto $\overrightarrow{u}$, what would the result be?

The result would be $\overrightarrow{0}$, since there is no component of either of the computed vectors in the direction of $\overrightarrow{u}$.

3. Project $\overrightarrow{v}$ onto $\overrightarrow{u}$.

$$\begin{aligned} \text{proj}_{\overrightarrow{u}}(\overrightarrow{v}) &= \frac{\overrightarrow{v} \cdot \overrightarrow{u}}{|\overrightarrow{u}|} \cdot \frac{\overrightarrow{u}}{|\overrightarrow{u}|} \\ &= \left(\frac{\langle 6, -1, 9 \rangle \cdot \langle -3, 7, 11 \rangle}{\langle -3, 7, 11 \rangle \cdot \langle -3, 7, 11 \rangle} \right) \langle -3, 7, 11 \rangle \\ &= \frac{74}{179} \langle -3, 7, 11 \rangle \end{aligned}$$