

Math 2283 - Introduction to Logic

Quiz #6 - 2015.09.14 Solutions

Consider the following theorem:

Theorem V: $y = x \wedge y = z \rightarrow z = x$

Using only the rule of detachment, substitution, and Laws III and IV given below, prove Theorem V.

Law III: $x = y \rightarrow y = x$

Law IV: $x = y \wedge y = z \rightarrow x = z$

First, we assume the hypothesis

(1) $y = x \wedge y = z$

An instance of L.o.A (BA) $p \wedge q \rightarrow p$ with $p : y = x, q : y = z$ gives

(2) $y = x \wedge y = z \rightarrow y = x$

R.o.D. with (1) and (2) gives

(3) $y = x$

An instance of L.o.A (BA) $p \wedge q \rightarrow q$ with $p : y = x, q : y = z$ gives

(4) $y = x \wedge y = z \rightarrow y = z$

R.o.D. with (1) and (4) gives

(5) $y = z$

An instance of Law III with $x : y$ and $y : z$

(6) $y = z \rightarrow z = y$

R.o.D. with (5) and (6) gives

(7) $z = y$

L.o.A (JT) $p \wedge q \rightarrow p \wedge q, p : z = y, q : y = x$

(8) $z = y \wedge y = x$

Instance of Law IV, with $x : z$ and $z : x$

(9) $z = y \wedge y = x \rightarrow z = x$

R.o.D. with (8) and (9) gives

(10) $z = x \quad \square$