

# Math 2283 - Introduction to Logic

Quiz #11 - 2015.10.19

Name: \_\_\_\_\_

---

The following is a ‘proof’ of the theorem:

If a relation  $R$  is antisymmetric, then it is also irreflexive.

There is something wrong with this proof, find it and correct it!

(1)  $R$  antisymmetric [ assume ]

(2)  $R$  antisymmetric  $\stackrel{\text{def}}{\longleftrightarrow}_{x,y} \mathbf{A} (xRy \rightarrow \sim yRx)$

(3)  $\mathbf{A}_{x,y} (xRy \rightarrow \sim yRx) \longleftrightarrow \mathbf{A}_{x,y} (\sim xRy \vee \sim yRx)$  [ instance of  $(p \rightarrow q) \longleftrightarrow (\sim p \vee q)$  ]

(4)  $\mathbf{A}_{x,y} (\sim xRy \vee \sim yRx) \longleftrightarrow \mathbf{A}_x (\sim xRx \vee \sim xRx)$  [ since universally quantified, let  $y = x$  ]

(5)  $\mathbf{A}_x (\sim xRx \vee \sim xRx) \longleftrightarrow \mathbf{A}_x (\sim xRx)$  [ instance of  $(p \vee p) \longleftrightarrow p$  ]

(6)  $\mathbf{A}_x (\sim xRx) \stackrel{\text{def}}{\longleftrightarrow} R$  irreflexive

(7)  $R$  irreflexive [ substitute (6) into (5) into (4) into (3) into (2) into (1) ]