Math 2143 - Brief Calculus with Applications

Exam #3 - 2014.11.30

Solutions

---

1. Compute the following integral:

$$\int x\sqrt{x^2+3}\,dx$$

If we let $u = x^2+3$, then $du = 2x\,dx$, or $x\,dx = du/2$. So we have

$$\begin{aligned}\int x\sqrt{x^2+3}\,dx &= \int \sqrt{u}\,\frac{1}{2}\,du \\ &= \frac{1}{2}\int u^{1/2}\,du \\ &= \frac{1}{2}\cdot\frac{2}{3}\,u^{3/2}+C \\ &= \frac{1}{3}\,(x^2+3)^{3/2}+C\end{aligned}$$

2. Compute the following derivative:

$$\frac{d}{dx}e^{\sqrt{x^2-3}}$$

$$\begin{aligned}\frac{d}{dx}e^{\sqrt{x^2-3}} &= e^{\sqrt{x^2-3}}\frac{d}{dx}\sqrt{x^2-3} \\ &= e^{\sqrt{x^2-3}}\cdot\frac{1}{2}\frac{1}{\sqrt{x^2-3}}\cdot\frac{d}{dx}(x^2-3) \\ &= e^{\sqrt{x^2-3}}\cdot\frac{1}{2}\frac{1}{\sqrt{x^2-3}}\cdot 2x \\ &= \frac{x\,e^{\sqrt{x^2-3}}}{\sqrt{x^2-3}}\end{aligned}$$

3. Compute the following integral:

$$\int \frac{(\ln(x))^3}{x}\,dx$$

Setting $u = \ln(x)$, then $du = dx/x$, which gives:

$$\begin{aligned}\int \frac{(\ln(x))^3}{x}\,dx &= \int u^3\,du \\ &= \frac{1}{4}\,u^4+C \\ &= \frac{1}{4}\,(\ln(x))^4+C\end{aligned}$$

4. Compute the following derivative:

$$\frac{d}{dx}\ln\left((5x+3)\cdot\sqrt{4x-5}\cdot\frac{7}{x^2+1}\right)$$

The first thing we do is simplify the expression we want to take the derivative of by using the rules of the logarithm:

$$\begin{aligned}\ln\left((5x+3)\cdot\sqrt{4x-5}\cdot\frac{7}{x^2+1}\right) &= \ln(5x+3)+\ln\left(\sqrt{4x-5}\right)+\ln\left(\frac{7}{x^2+1}\right) \\ &= \ln(5x+3)+\frac{1}{2}\ln(4x-5)+\ln(7)-\ln\left(x^2+1\right)\end{aligned}$$

Now we take a derivative:

$$\frac{d}{dx}\ln\left((5x+3)\cdot\sqrt{4x-5}\cdot\frac{7}{x^2+1}\right) = \frac{d}{dx}\left[\ln(5x+3)+\frac{1}{2}\ln(4x-5)+\ln(7)-\ln\left(x^2+1\right)\right]$$
$$= \frac{5}{5x+3}+\frac{1}{2}\cdot\frac{4}{4x-5}+0-\frac{2x}{x^2+1}$$

5. Compute the following integral:

$$\int \frac{x}{\sqrt{x^2-3}}\, e^{\sqrt{x^2-3}}\, dx$$

Most likely, the only substitution worth doing is $u = \sqrt{x^2-3}$, which results in $du = \frac{x}{\sqrt{x^2-3}}\, dx$ (by chain rule), which is very convenient!

$$\int \frac{x}{\sqrt{x^2-3}}\, e^{\sqrt{x^2-3}}\, dx = \int e^u\, du$$
$$= e^u + C$$
$$= e^{\sqrt{x^2-3}} + C$$

6. Find the area between the curve $y = \sqrt{x}$ and the x-axis for $0 \leq x \leq 1$.

Area is given by:

$$\int_0^1 \sqrt{x}\, dx = \frac{2}{3}x^{3/2}\Big|_0^1$$
$$= \frac{2}{3}(1-0)$$
$$= \frac{2}{3}$$

7. Find the area between the curve $y = \sqrt{x}$ and $y = x$ using your answer to problem 6 and WITHOUT doing any further integrations. (Hint: You can geometrically find the area under $y = x$ for $0 \leq x \leq 1$.)

The area under $y = x$ is the area of the triangle with length 1 and height 1. So the area between the two curves is $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.