

Math 2143 - Brief Calculus with Applications

Final Exam - 2015.12.07

Name: _____

1. Compute the following limits:

[5 pts] (a) $\lim_{x \rightarrow 1} \frac{\sqrt{3x+2} - \sqrt{5}}{x-1}$

[5 pts] (b) $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x + 1}$

[5 pts] (c) $\lim_{h \rightarrow 0} \frac{\frac{1}{h^2 + 1} - 1}{h}$

2. Compute the following derivatives:

[7 pts] (a) $\frac{d}{dx} \frac{\ln(3x-2)e^{4x+5}}{1+3x^2}$

[7 pts] (b) $\frac{d}{dx} \ln(x^2 + e^{4x} - 1)$

[7 pts] (c) $\frac{d}{dx} e^{x/(x^2+1)}$

[7 pts] (d) $\frac{d}{dx} \sqrt{\frac{3x+1}{3x-1}}$

3. [10 pts] Find the equation of the tangent line to $f(x) = \frac{e^x + e^{-x}}{2}$ at $x = 0$.

4. [10 pts] Find dy/dx if $1 + e^{xy} = x^2 + y^2$

5. For this problem, consider $f(x) = \frac{x^2}{x+1}$.

[3 pts] (a) What is the domain of $f(x)$?

[3 pts] (b) Locate the roots of $f(x)$.

[4 pts] (c) Determine the locations of all vertical asymptotes.

[4 pts] (d) Determine the locations of all horizontal/slant asymptotes.

[5 pts] (e) Compute $f'(x)$. Be sure to simplify your answer.

[5 pts] (f) Determine the critical points of $f(x)$ using $f'(x)$.

[5 pts] (g) Compute the intervals of increase and decrease using $f'(x)$.

[4 pts] (h) Classify the critical points as local maximums and minimums.

[5 pts] (i) Compute $f''(x)$. Be sure to simplify your answer.

[5 pts] (j) Determine intervals of concavity.

[4 pts] (k) Locate all inflection points.

[10 pts] (l) Sketch the graph of $f(x)$ using all of the information from parts (a)–(k).

6. Compute the following integrals:

[7 pts] (a) $\int \frac{3e^{3x} - 1}{e^{3x} - x} dx$

[7 pts] (b) $\int \frac{x^4 - x^{1/3} + x^{2/5} - 1}{x^2} dx$

[7 pts] (c) $\int_{-1}^1 x^3 - x^5 + 3x \, dx$

[7 pts] (d) $\int \ln(x) \, dx$