

Math 2143 - Brief Calculus with Applications

Practice Final Exam - 2015.11.30

Name: _____

1. Compute the following limits:

[5 pts] (a) $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

[5 pts] (b) $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x - 4}$

[5 pts] (c) $\lim_{x \rightarrow 1} \frac{\frac{1}{x^2 + 1} - \frac{1}{2}}{x - 1}$

2. Compute the following derivatives:

[7 pts] (a) $\frac{d}{dx} \frac{x}{x^2 + 1}$

[7 pts] (b) $\frac{d}{dx} \sqrt{\frac{x}{x^2 + 1} - 3x^2 + 2}$

[7 pts] (c) $\frac{d}{dx} (e^{3x+1} \ln(x^2 - 1) (x^3 + 3x^2 - 1))$

[7 pts] (d) $\frac{d}{dx} \ln(\ln(\ln(x)))$

3. [10 pts] Find the equation of the tangent line to $f(x) = 3x^2 + 4x - 3 + \frac{2}{x^2}$ at $x = 1$.

4. [10 pts] Find dy/dx if $\frac{x+y}{y-2x} = e^{x^2y}$

5. [15 pts] Find the absolute maximum and minimum of the function $f(x) = x^3 + \frac{1}{2}x^2 - 2x + 5$ on the interval $[-2, 1]$.

6. For this problem, consider $f(x) = \frac{(x-2)(x+1)}{x^2}$.

[3 pts] (a) What is the domain of $f(x)$?

[3 pts] (b) Locate the roots of $f(x)$.

[4 pts] (c) Determine the locations of all vertical asymptotes.

[4 pts] (d) Determine the locations of all horizontal asymptotes.

[5 pts] (e) Compute $f'(x)$. Be sure to simplify your answer.

[5 pts] (f) Determine the critical points of $f(x)$ using $f'(x)$.

[5 pts] (g) Compute the intervals of increase and decrease using $f'(x)$.

[4 pts] (h) Classify the critical points as local maximums and minimums.

[5 pts] (i) Compute $f''(x)$. Be sure to simplify your answer.

[5 pts] (j) Determine intervals of concavity.

[4 pts] (k) Locate all inflection points.

[10 pts] (l) Sketch the graph of $f(x)$ using all of the information from parts (a)–(k).

7. Compute the following integrals:

[7 pts] (a) $\int xe^{3x^2-2x+1} dx$

[7 pts] (b) $\int x^2e^{3x-2x+1} dx$

[7 pts] (c) $\int 3x\sqrt{x^2-1} dx$

[7 pts] (d) $\int \frac{x^3 + 2x - 1}{x^2} dx$

[7 pts] (e) $\int \frac{1}{x \ln(x)} dx$

8. [10 pts] Find the finite area between the curves $f(x) = x^3 - 1$ and $g(x) = x^2 - 1$.

9. Consider a function $C(t)$ where t is measured in seconds and C in degrees Celsius per second.

[5 pts] (a) What are the units on $C'(t)$?

[5 pts] (b) What are the units on $\int C(t) dt$?

[5 pts] (c) What does $\int_0^{60} C(t) dt$ represent physically?